Carroll limits and flat space holography

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Based mainly on 2207.03468 (SciPost Phys.) with Stefano Baiguera, Watse Sybesma and Benjamin Søgaard

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Outline

- Why not Lorentzian?
- Newton-Cartan, Carroll and flat space holography
- Constructing Carroll CFTs
- Outlook

Why not Lorentzian?

What's wrong with Lorentzian symmetry?
Nothing, but *general relativity is hard!*

Einstein gravity contains Newtonian gravity,

$$g_{00} = -(1 + 2\Phi), \quad v/c \ll 1, \quad \text{weak coupling} \;\; G \ll 1$$

but where is the geometry?

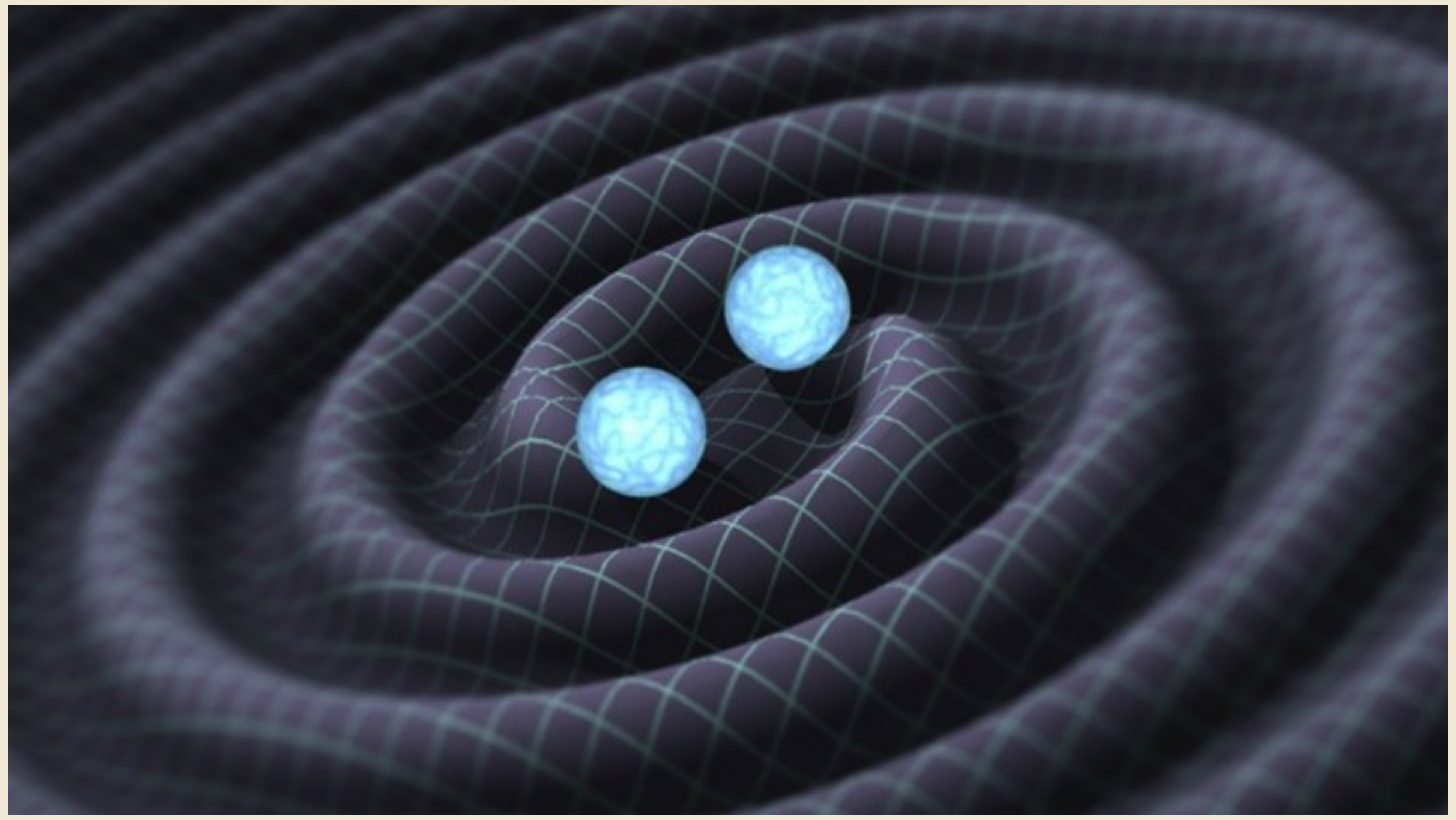
$\Longrightarrow$ Newton-Cartan geometry! [Cartan] [Künzle] [Dautcourt] …

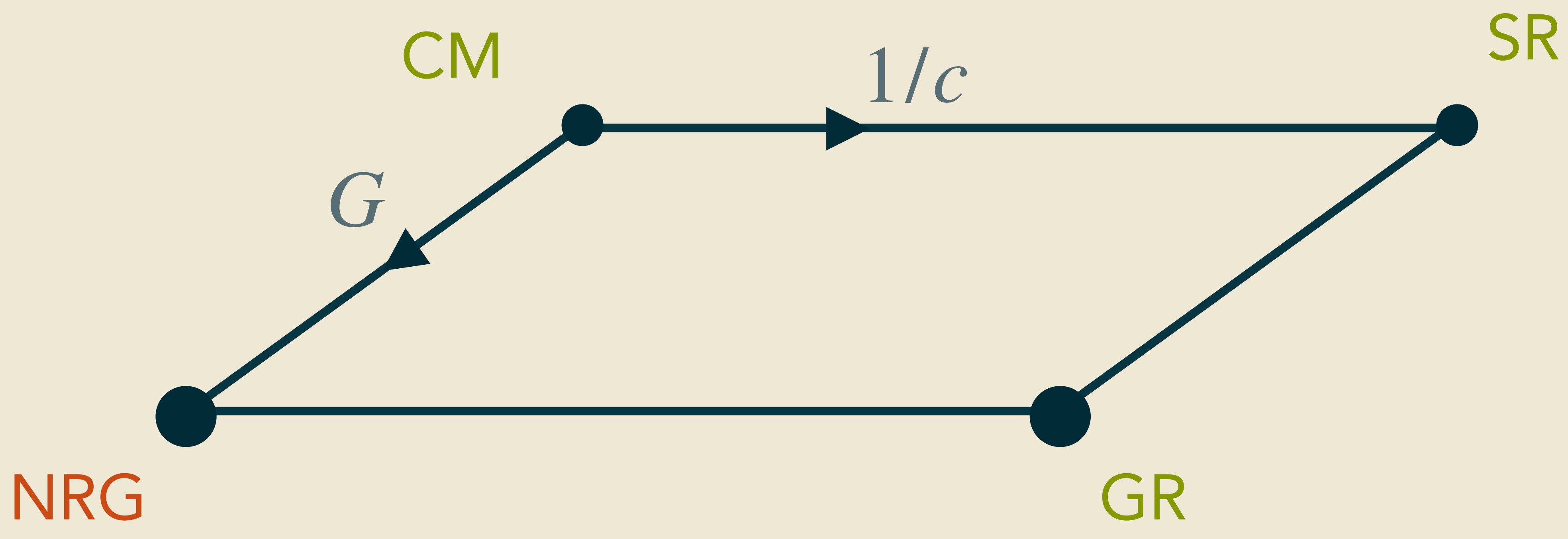
Now understand better [Van den Bleeken] [Hansen, Hartong, Obers]

- how Newton-Cartan geometry arises from Lorentzian
- how Newtonian gravity arises from GR
- weak coupling and low velocity are independent!

Main tool: covariant expansion of geometry in powers of c

[Niels', Jelle's and Jørgen's talk], see also review [Hartong, Obers, GO]





Why not Lorentzian?

What's wrong with Lorentzian symmetry?
Nothing, but *string theory is also hard!*

Simpler subsector: non-relativistic strings and quantum gravity
[Gomis, Ooguri] [Danielsson, Guijosa, Kruczenski]

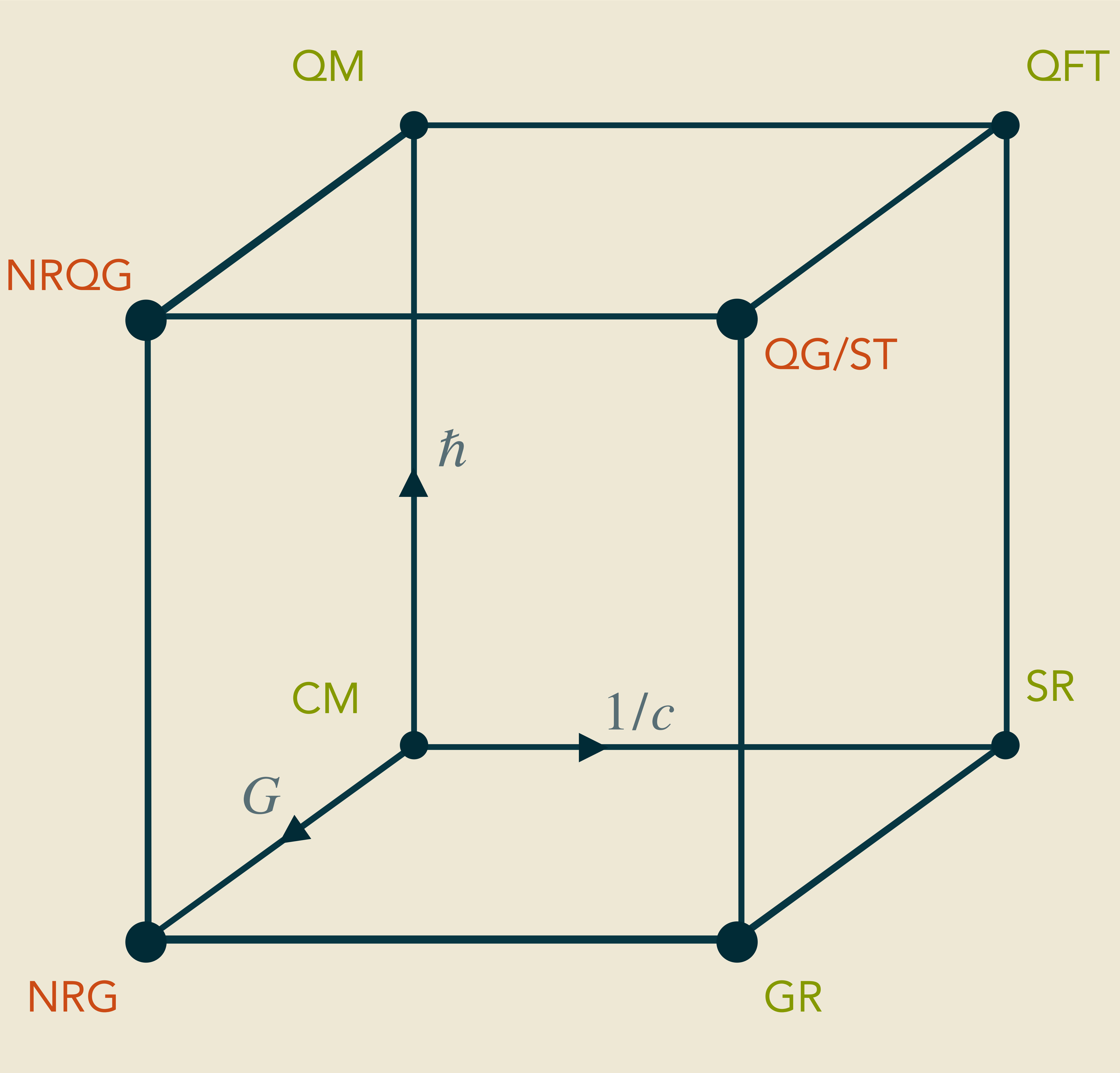
- decoupling limit of string theory
- non-relativistic spectrum
- easier worldsheet theory?

[Matthias' poster], see also review [GO, Yan]

Access non-relativistic regimes of AdS/CFT? [Troels' talk]

This talk: non-Lorentzian geometry for flat space holography?

ultra-local $c \to 0$ Carroll limit



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Newton-Cartan and Carroll geometry

compare: Lorentzian geometry

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -f\,dt^2 + f^{-1} dr^2 + r^2 d\Omega_2$$

Compatible connection $\nabla_\rho g_{\mu\nu} = 0$

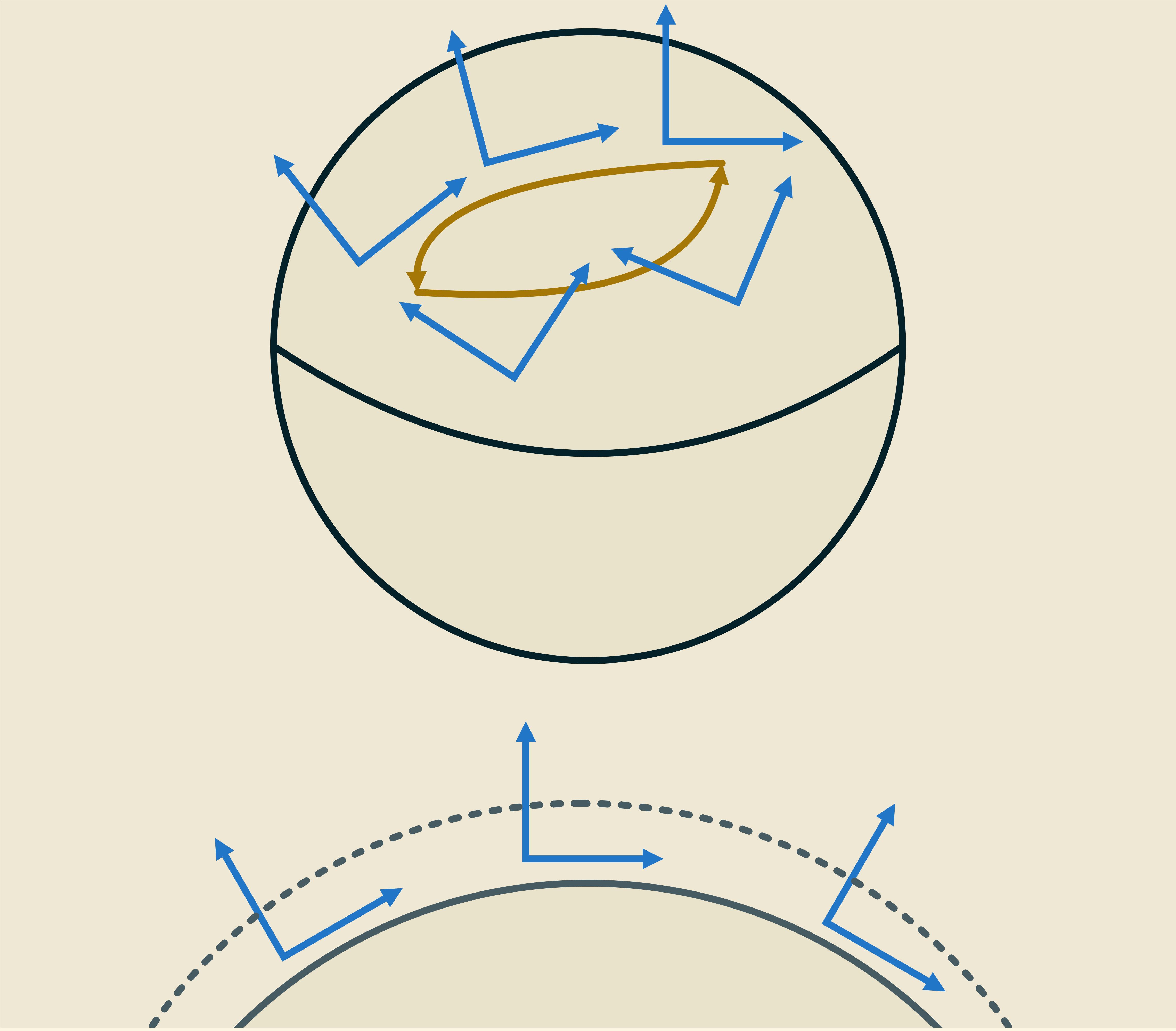
defines curvature $[\nabla_\mu, \nabla_\nu]\, X^\sigma = -R_{\mu\nu\rho}{}^\sigma X^\rho$

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}_{AB} \begin{pmatrix} \sqrt{f} & 0 \\ 0 & 1/\sqrt{f} \end{pmatrix}_\mu{}^A \begin{pmatrix} \sqrt{f} & 0 \\ 0 & 1/\sqrt{f} \end{pmatrix}_\nu{}^B$$

$$= \eta_{AB}\, e_\mu{}^A e_\nu{}^B$$

metric has local Minkowski structure

Mirror this for local Galilean and local Carroll structures



Newton-Cartan geometry

Galilean boost

$$\begin{pmatrix} t \\ x \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ v & 1 \end{pmatrix} \begin{pmatrix} t' \\ x' \end{pmatrix}$$

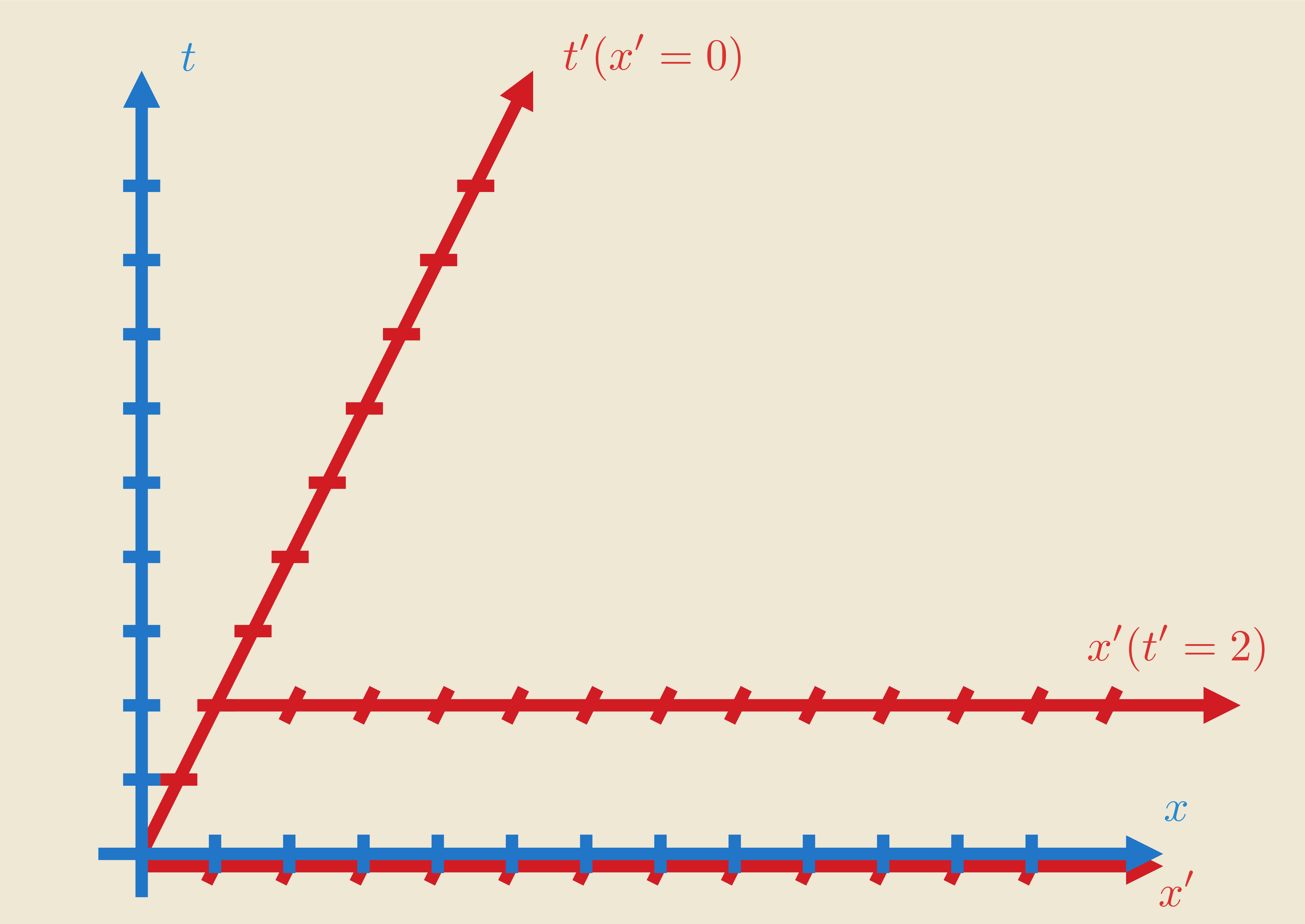
preserves

- time coordinate $\begin{pmatrix} 1 & 0 \end{pmatrix}$
- space direction $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

For curved geometry:

- clock one-form $\tau_\mu dx^\mu \sim \begin{pmatrix} 1 & 0 \end{pmatrix}$
- spatial cometric $h^{\mu\nu}\partial_\mu\partial_\nu \sim$ twice $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

known as Newton-Cartan geometry



Newton-Cartan geometry

Newton-Cartan geometry $\tau_\mu(x^\rho)$ and $h^{\mu\nu}(x^\rho)$

Has local Galilean structure!

$$\tau_\mu \sim (1 \quad 0) \quad \text{and} \quad h^{\mu\nu} = \delta^{ab} e^\mu{}_a e^\nu{}_b \sim \text{twice} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

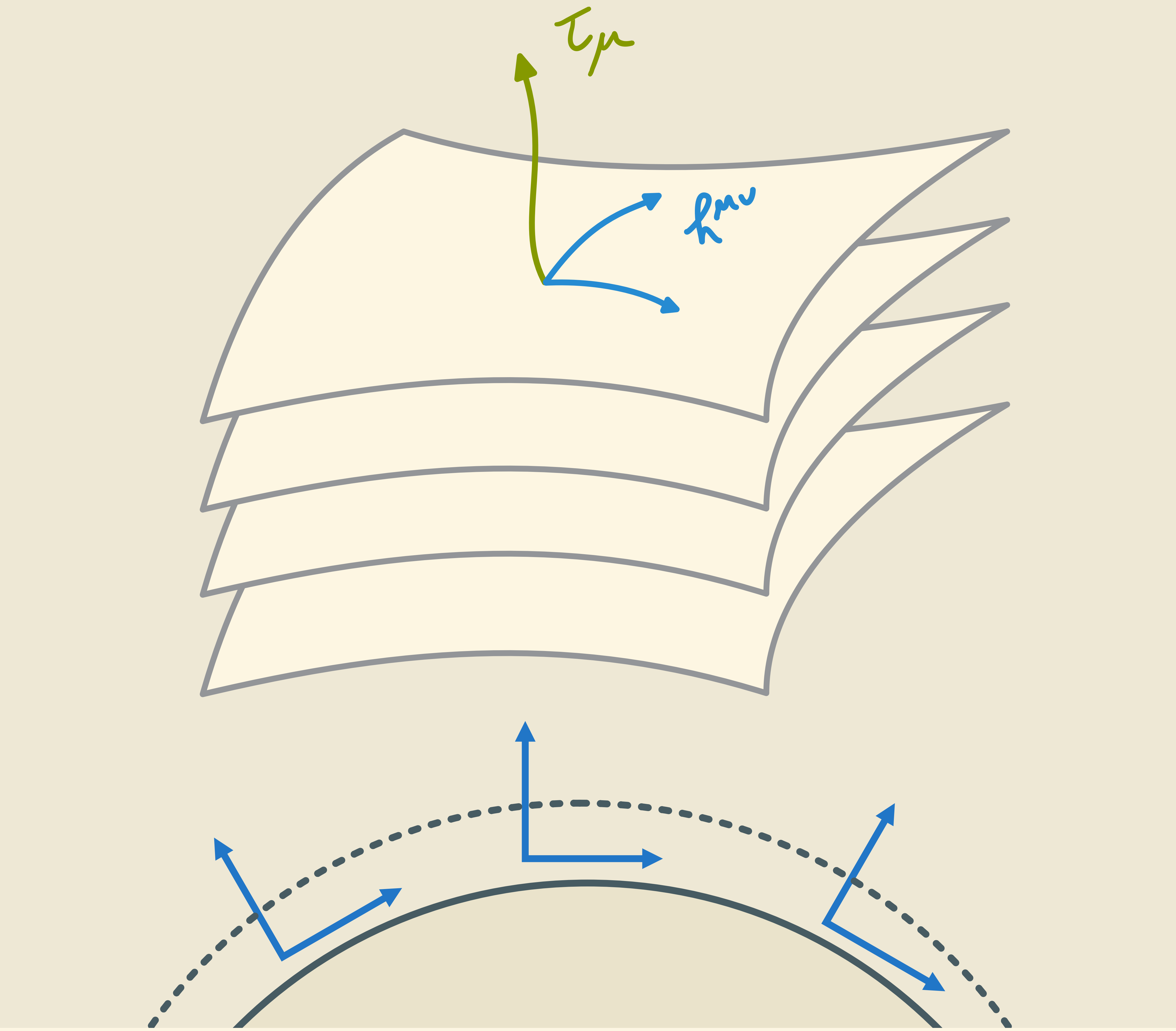
Clock form defines spatial foliation (if $\tau \wedge d\tau = 0$), e.g.

$$\tau_\mu dx^\mu = -\sqrt{1 - \frac{R}{r}} dt \,, \quad h^{\mu\nu}\partial_\mu \partial_\nu = \left(1 - \frac{R}{r}\right) \partial_r^2 + \frac{1}{r^2} \partial_{\Omega_2}$$

Compatible connection $\check{\nabla}_\rho \tau_\mu = 0$ and $\check{\nabla}_\rho h^{\mu\nu} = 0$

curvature $[\check{\nabla}_\mu, \check{\nabla}_\nu]\, X^\sigma = -\check{R}_{\mu\nu\rho}{}^\sigma X^\rho$

torsion $2\check{\Gamma}^\rho_{[\mu\nu]} = 2\tau^\rho \partial_{[\mu} \tau_{\nu]}$ determined by $d\tau$



Carroll geometry

Carroll boost

$$\begin{pmatrix} t \\ x \end{pmatrix} = \begin{pmatrix} 1 & v \\ 0 & 1 \end{pmatrix} \begin{pmatrix} t' \\ x' \end{pmatrix}$$

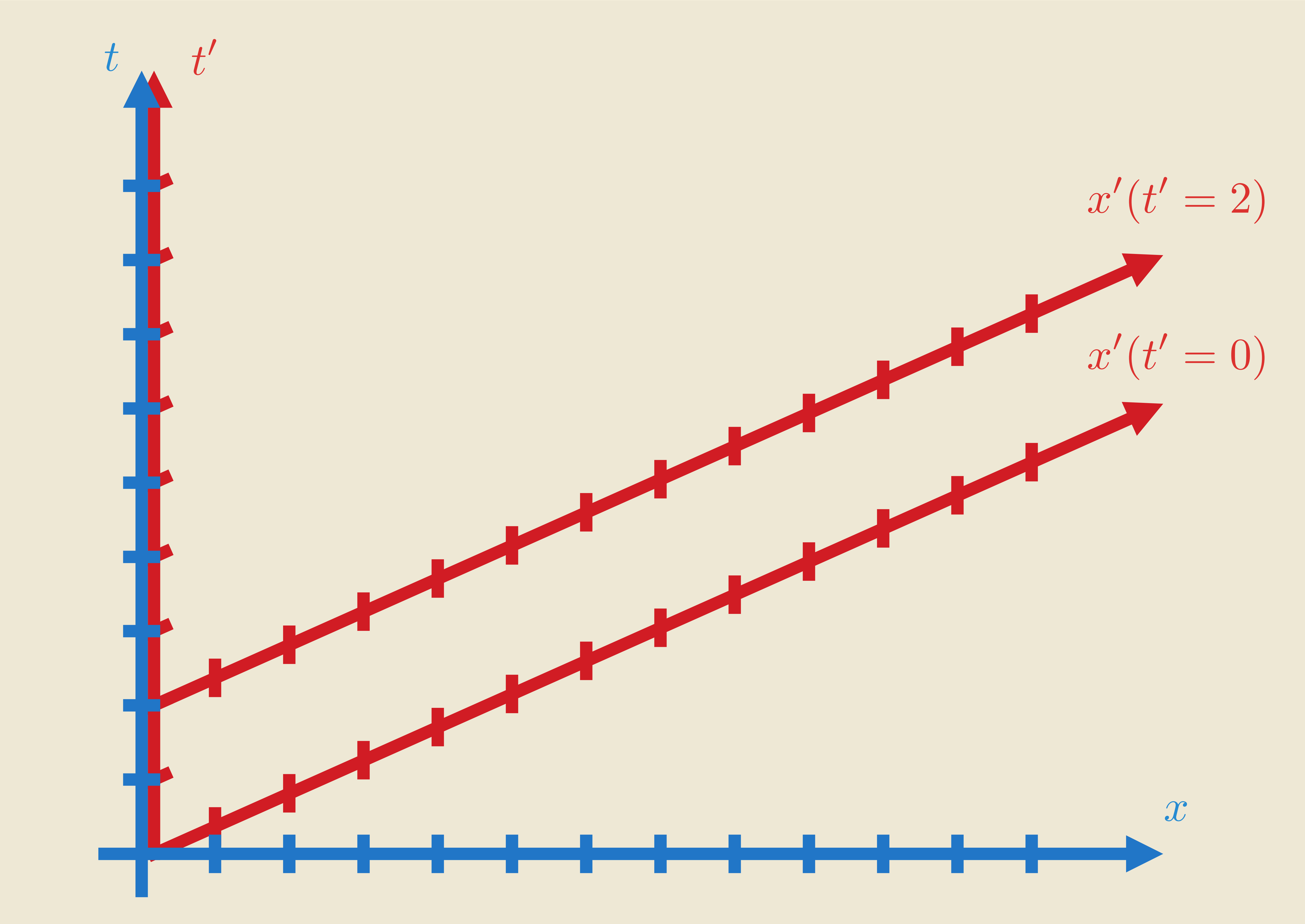
preserves

- time direction $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
- space coordinate $\begin{pmatrix} 0 & 1 \end{pmatrix}$

For curved geometry:

- time vector field $v^\mu \partial_\mu \sim \begin{pmatrix} 1 \\ 0 \end{pmatrix}$
- spatial metric $h_{\mu\nu} dx^\mu dx^\nu \sim$ twice $\begin{pmatrix} 0 & 1 \end{pmatrix}$

known as Carroll geometry



Carroll geometry

Starting from `relativistic' Lorentz boosts

$$t \to t + \beta x, \qquad x \to x + \beta t$$

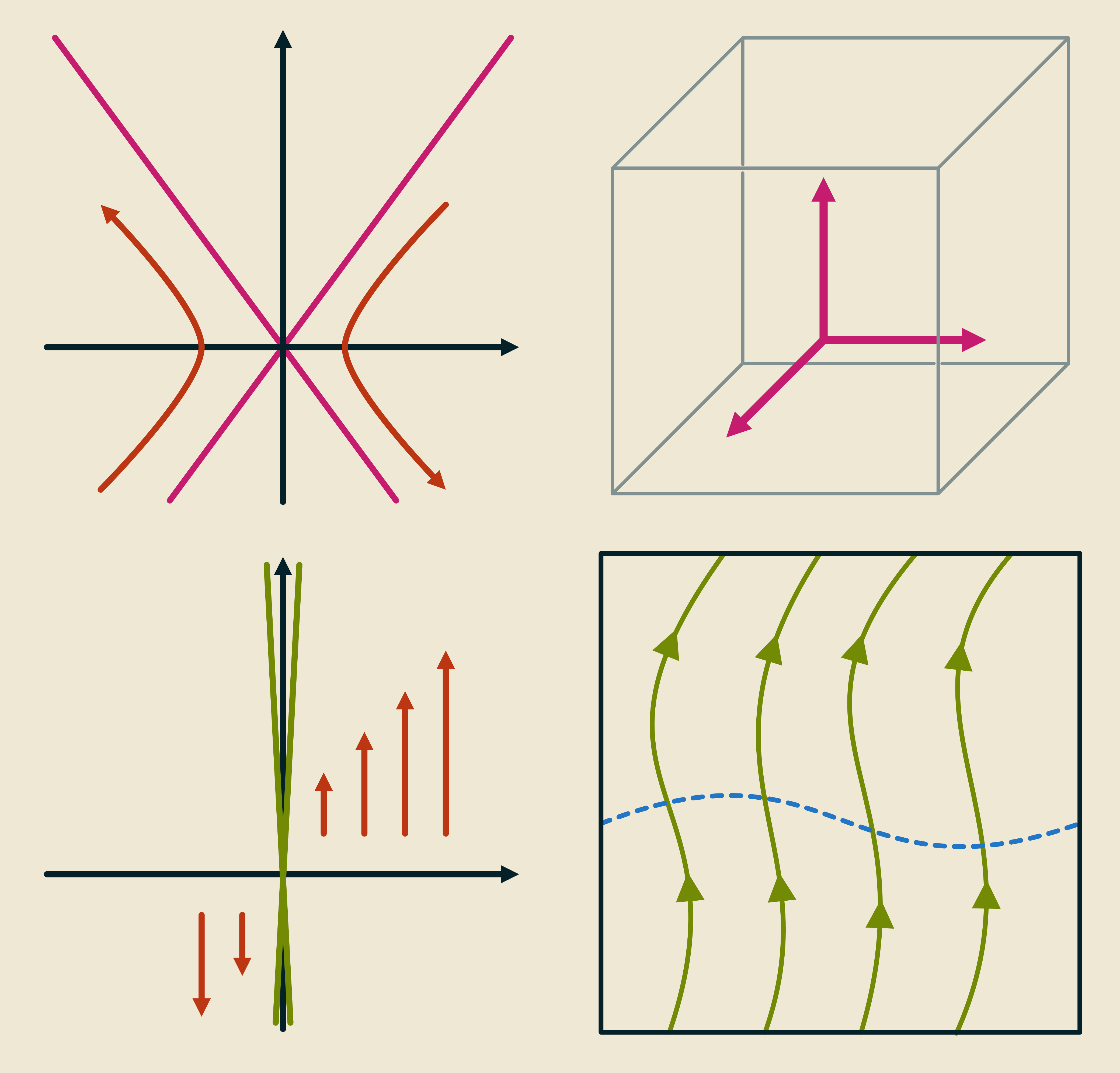
get Carroll boosts in ultra-local limit $c \to 0$, [Levy-Leblond] [Sen Gupta]

$$t \to t + \lambda x\,, \quad x \to x \quad \text{and} \quad \partial_t \to \partial_t\,, \quad \partial_x \to \partial_x + \lambda \partial_t$$

Spatial metric $h_{\mu\nu}(x^\rho)$ and time vector field $v^\mu(x^\rho)$
in contrast to Newton-Cartan $h^{\mu\nu}(x^\rho)$ and $\tau_\mu(x^\rho)$

Less obviously physical, but

- ultra-local behavior leads to solvable systems [Niels' talk]
- appears in Lorentzian geometry on null surfaces such as $\mathscr{I}^+$
- BMS asymptotic symmetries are isomorphic
  to conformal Carroll algebra [Duval, Gibbons, Horvathy, Zhang]



Carroll geometry and flat holography

Holographic dual field theory for asymptotically flat spacetimes?

In 3+1 dim: BMS_4 asymptotic symmetries on $\mathscr{I}^+ \simeq \mathbb{R} \times S^2$

superrotations $z \to g(z), \quad \bar{z} \to \bar{g}(\bar{z})$

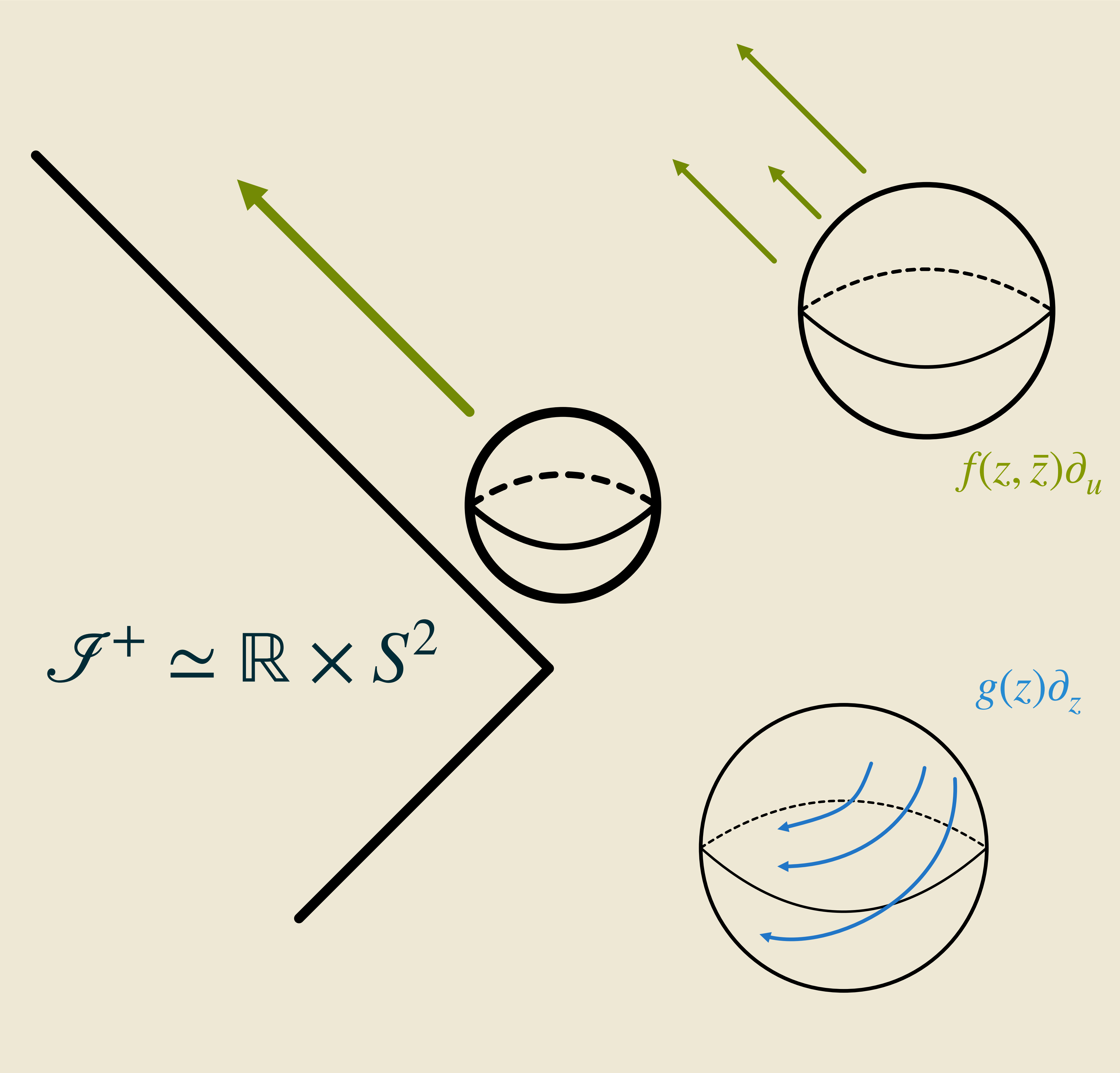
- Virasoro symmetries of CFT_2
- suggests 2d celestial CFT dual: CCFT_2

supertranslations $u \to u + f(z, \bar{z})$

- $\sim$ Carroll boosts at each $(z, \bar{z})$
- suggests 3d Carrollian CFT dual: $\text{BMS}_4 \simeq \text{CCar}_3$

See also [Donnay, Fiorucci, Herfray, Ruzziconi] and [Bagchi, Banerjee, Basu, Dutta]

Few explicit CCFT_2 theories known,
but *can construct CCar_3 examples from $c \to 0$ limit!*



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- Discussion and outlook

Main goal: find explicit actions for conformal Carroll theories

- Discuss consequences of Carroll boosts in field theory

- Use limits to obtain conformal Carroll action from

$$S = -\frac{1}{2}\int d^dx\sqrt{-g}\left(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{d-2}{4(d-1)}R\phi^2\right)$$

Carroll geometry

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Geometry from spatial metric $h_{\mu\nu}(x^\rho)$ and time vector field $v^\mu(x^\rho)$

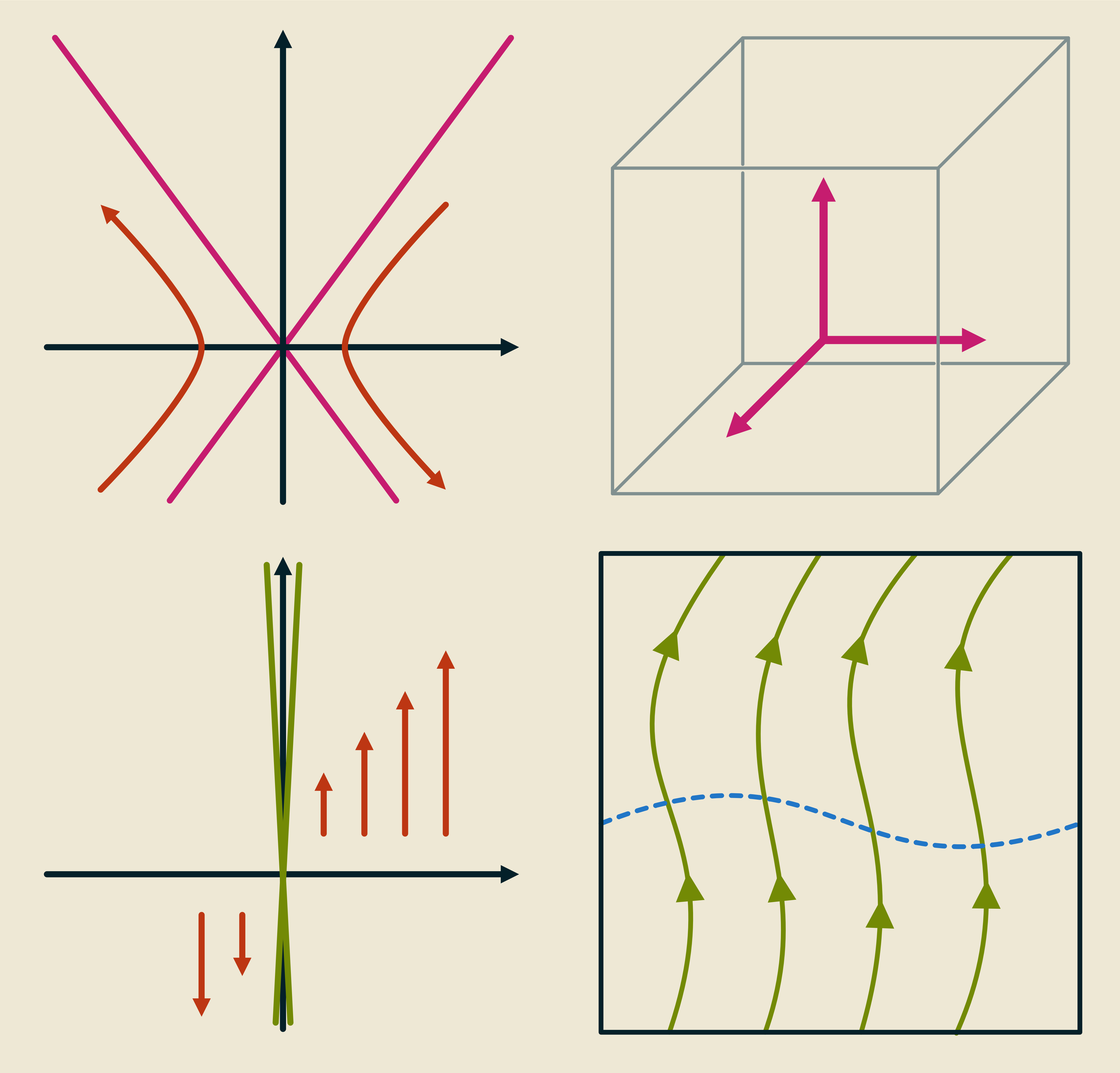
Complement with inverse $\tau_\mu(x^\rho)$ and $h^{\mu\nu}(x^\rho)$, satisfy

$$v^\mu h_{\mu\nu} = 0, \quad \tau_\mu h^{\mu\nu} = 0, \quad v^\mu \tau_\mu = -1, \quad \delta^\mu_\nu = -v^\mu \tau_\nu + h^{\mu\rho} h_{\rho\nu}$$

Transform under local Carroll boosts $\lambda_\mu(x^\rho)$ as

$$\delta_\lambda \tau_\mu = \lambda_\mu\,, \qquad \delta_\lambda h^{\mu\nu} = \lambda^\mu v^\nu + v^\mu \lambda^\nu$$

[Duval, Gibbons, Horvathy, Zhang] [Hartong] [Ciambelli, Marteau, Petropoulos…]
[Hansen, Obers, GO, Søgaard] …

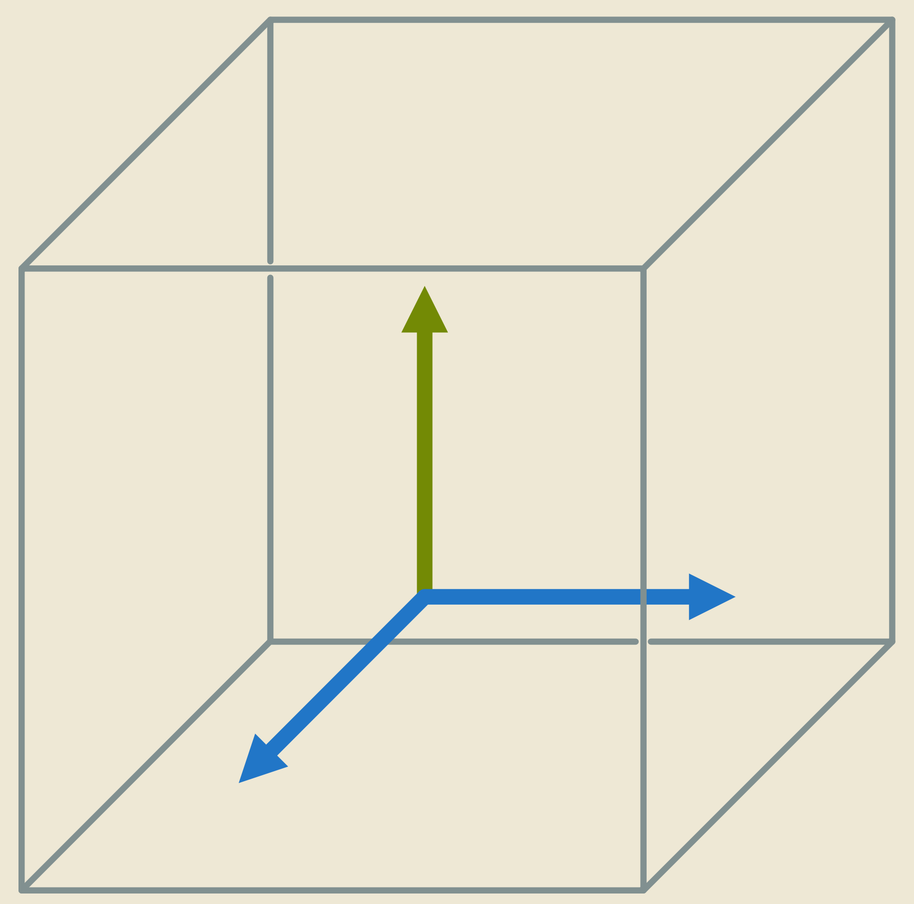


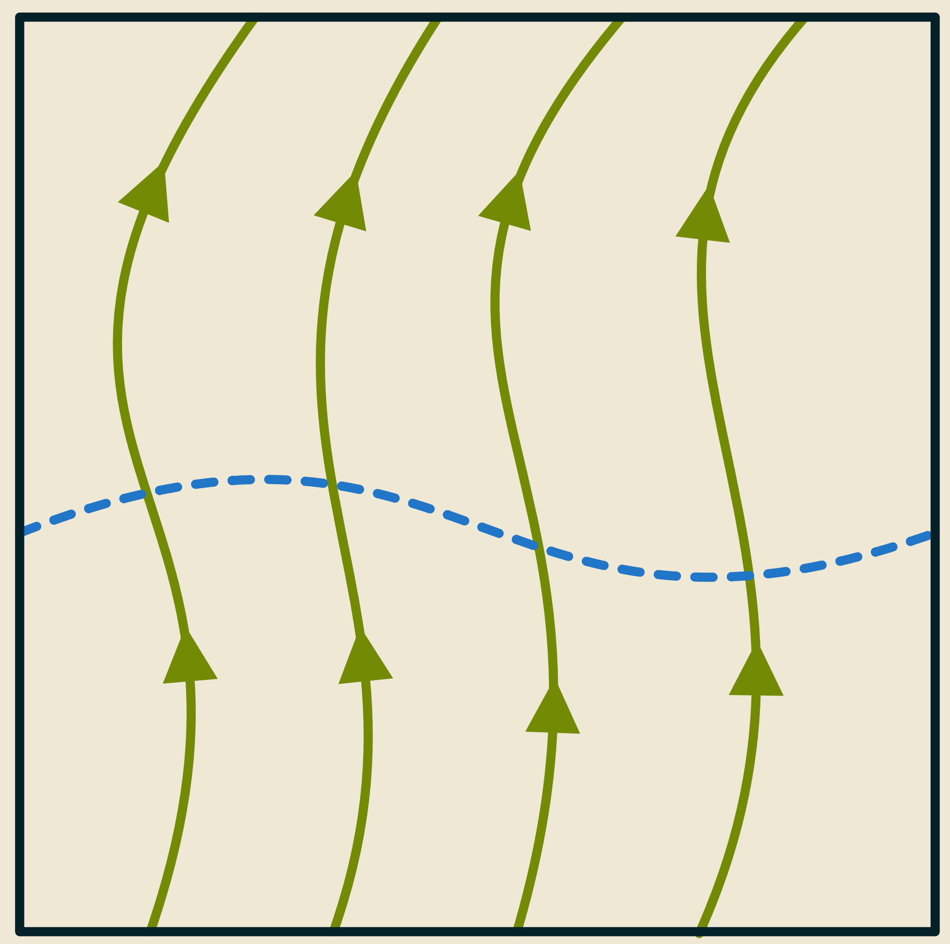
Carroll geometry

Local Carroll boost symmetry

- inevitable for limit of Lorentz-invariant theory
- implies vanishing energy flux $T^i{}_0 = 0$
- 'timelike' or 'spacelike' $\langle \phi(u,z,\bar{z})\, \phi(0,0,0) \rangle = \begin{cases} f(u)\delta^{(2)}(z,\bar{z}) \\ g(z) \end{cases}$

[Henneaux, Salgado-Rebodello] [De Boer, Hartong, Obers, Sybesma, Vandoren]





Conformal scalar actions: timelike

Consider Lorentzian conformal scalar action,

$$S = -\frac{1}{2}\int d^d x \sqrt{-g}\left(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{d-2}{4(d-1)}R\phi^2\right)$$

In Carroll limit $c \to 0$, leading-order terms give [Baiguera, GO, Sybesma, Søgaard]

$$S_t = -\frac{1}{2}\int d^d x\, e\left[-(v^\mu\partial_\mu\phi)^2 + \frac{(d-2)}{4(d-1)}\left(K^{\mu\nu}K_{\mu\nu} + K^2 - 2v^\mu\partial_\mu K\right)\phi^2\right]$$

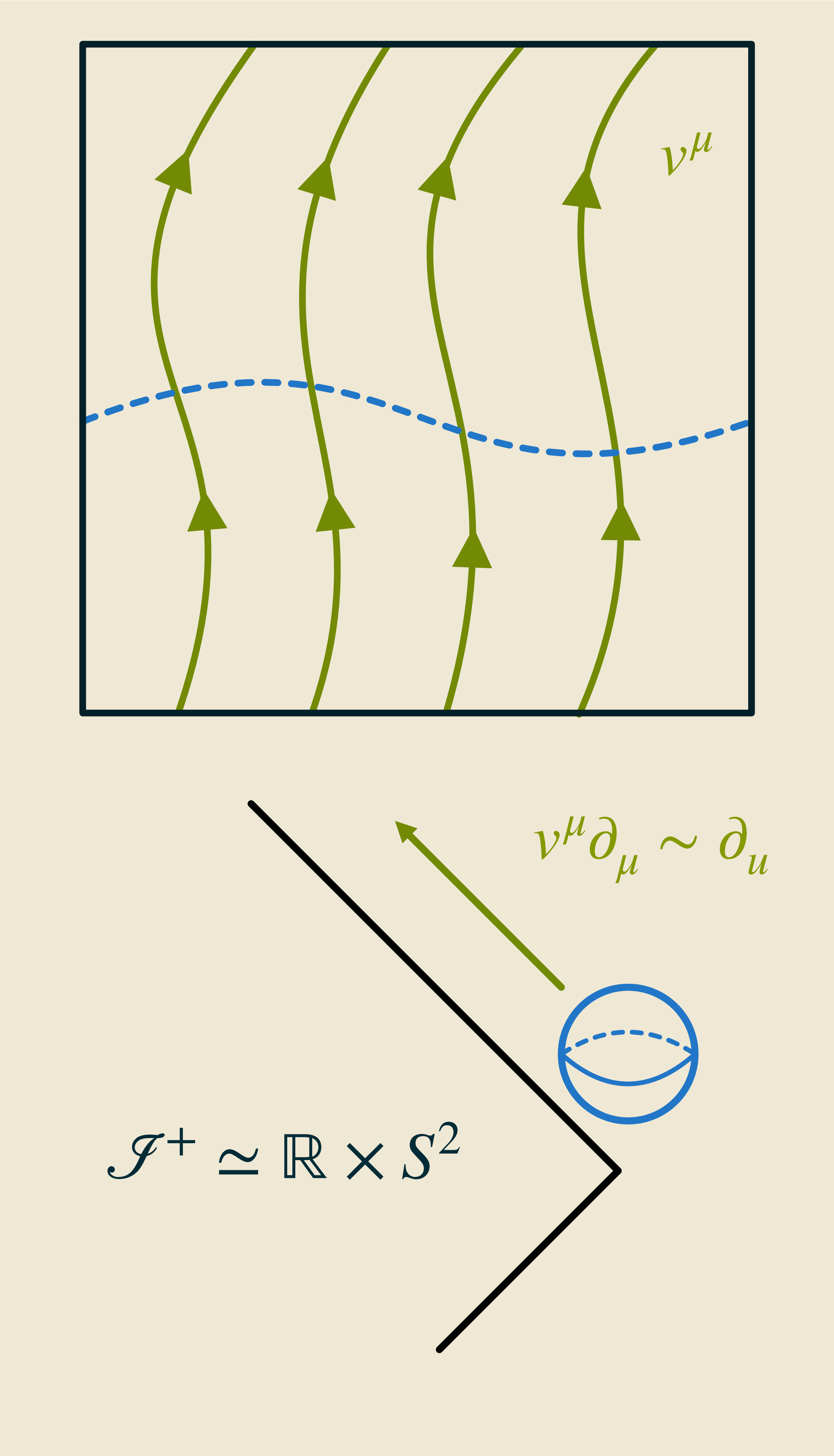
where $K_{\mu\nu} = -\frac{1}{2}\mathscr{L}_v h_{\mu\nu}$ is extrinsic curvature

This is timelike conformal Carroll scalar, flat space propagator $\sim u\,\delta^{(2)}(z,\bar{z})$

Carroll boost-invariant and Weyl-invariant, so ${T^i}_0 = 0$ and ${T^\mu}_\mu = 0$

Also considered from no-boost approach in [Gupta, Suryanarayana] [Rivera-Betancour, Vilatte]

Reproduces celestial CCFT correlators [Bagchi, Banerjee, Basu, Dutta]



Conformal scalar actions: spacelike

Consider Lorentzian conformal scalar action,

$$S = -\frac{1}{2}\int d^d x \sqrt{-g}\left(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{d-2}{4(d-1)}R\phi^2\right)$$

Alternative Carroll limit $c \to 0$ gives

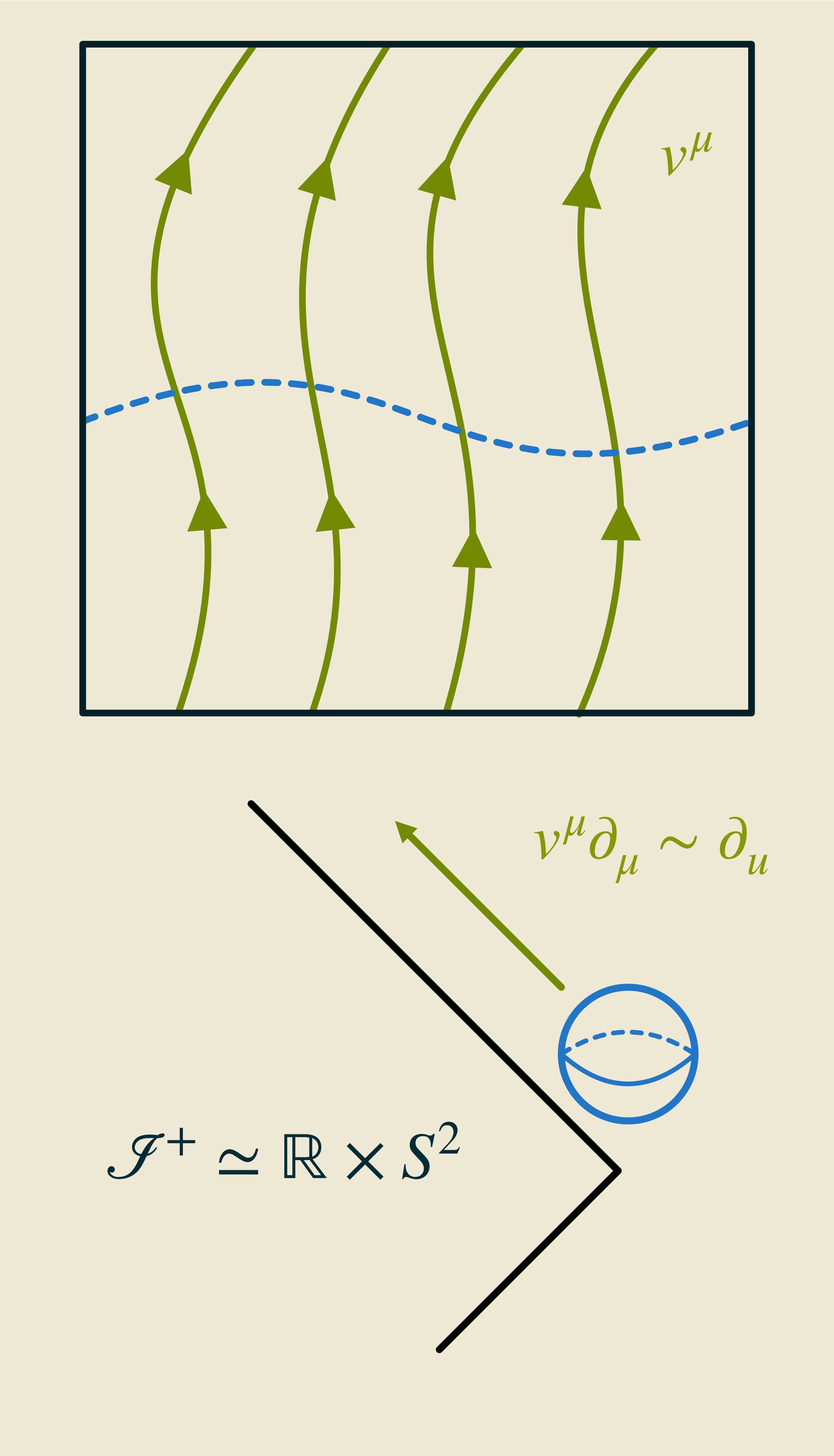
$$S_s = -\frac{1}{2}\int d^d x\, e\left[h^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{(d-2)}{4(d-1)}\left(h^{\mu\nu}\tilde{R}_{\mu\nu} - \tilde{\nabla}_\mu a^\mu\right)\right]$$

together with two constraints

- time-dependence fixed by $v^\mu\partial_\mu\phi = -\frac{(d-2)}{4(d-1)}K$
- extrinsic curvature must be pure trace $K_{\mu\nu} = \frac{h_{\mu\nu}}{d-1}K$

This is spacelike conformal Carroll scalar. [Baiguera, GO, Sybesma, Søgaard]

Boost- and Weyl-invariant, flat space propagator $\sim \log(x)^2$ spacelike



Conformal scalar actions: spacelike

Can dimensionally reduce spacelike action

$$S_s \implies -\frac{1}{2}\int d^{d-1}x\sqrt{h}\,\left(h^{\mu\nu}\partial_\mu\hat{\phi}\partial_\nu\hat{\phi} + \frac{(d-3)}{4(d-2)}\hat{R}\hat{\phi}^2 + \frac{1}{4(d-1)(d-2)}A^{-2}\hat{R}_{A^{-2}h_{ij}}\right)$$

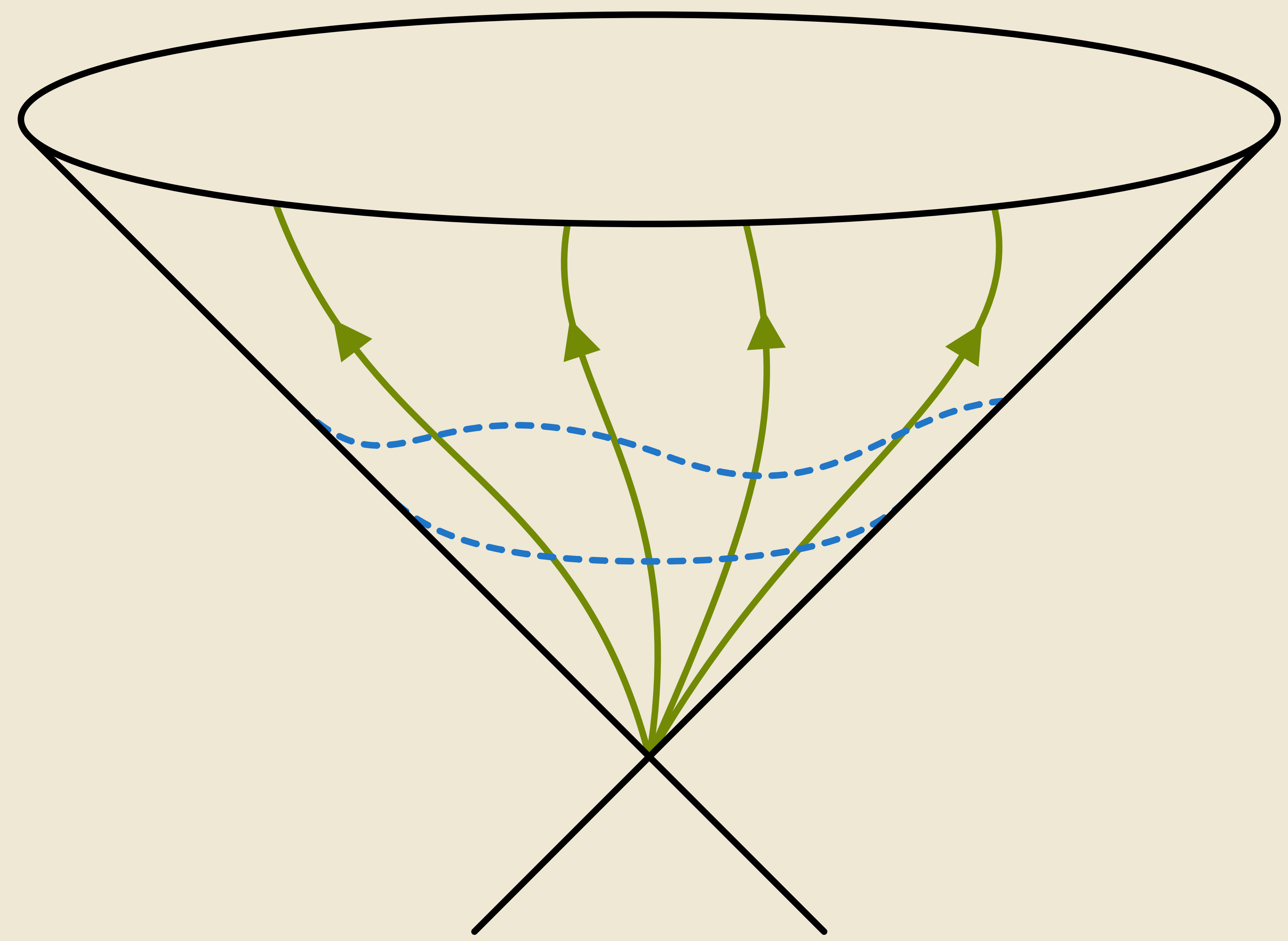
Reminiscent of embedding space formalism!

Get $(d-1)$-dim conformal $SO(d,1)$ representations
from $(d+1)$-dim Lorentz representations in $\mathbb{R}^{1,d}$

Restriction to light cone
$\implies$ Carrollian spacelike theory
$\implies$ Euclidean CFT$_{d-1}$ theory

Similar procedure for other spacelike Carroll theories?

Application to non-vacuum correlators in CFT$_{d-1}$
from 'light-cone Fefferman-Graham'? [Parisini, Skenderis, Withers]



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To boost or not to boost?

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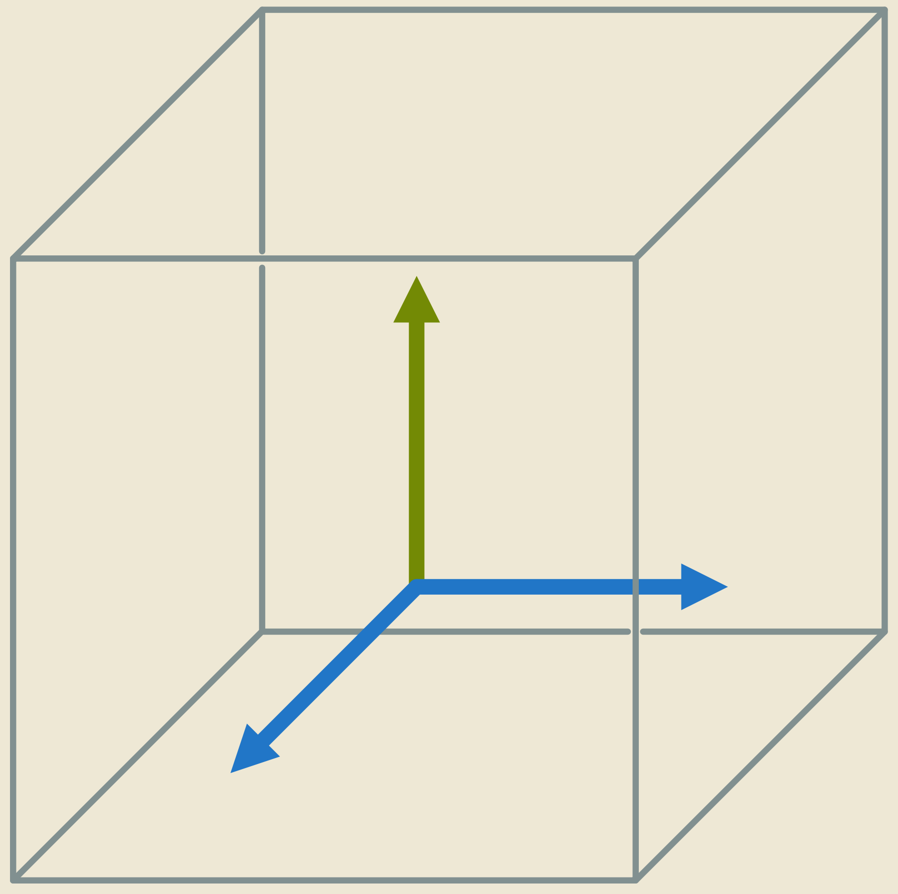


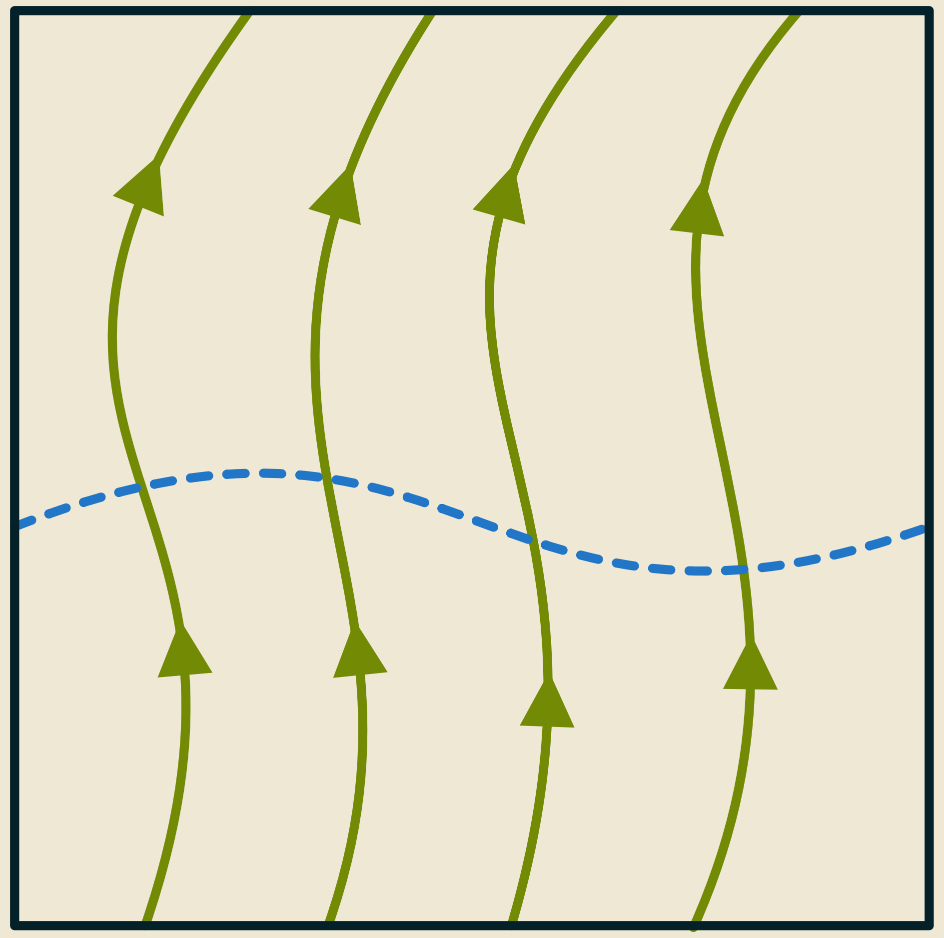
However maybe *Carroll boosts not always desired* in flat space holography?

- known holographic fluids with $T^i{}_0 \neq 0$ [Ciambelli, Marteau, Petkou, Petropoulos, Siampos]
- focus instead on $(v^\mu, h_{\mu\nu})$ fiber structure?

[Ciambelli, Leigh, Marteau, Siampos] [Petkou, Petropoulos, Rivera Betancour, Siampos] [Freidel, Jai-akson]…

- go to Lorentz-breaking frame before taking flat/Carroll limit in AdS/CFT?

cf [Campoleoni, Ciambelli, Delfante, Marteau, Petropoulos, Ruzziconi]



To boost or not to boost?

Breaking Carroll boost ~ breaking supertranslation symmetry in celestial holography

For massless particles with $p^\mu = \omega\, q^\mu(z,\bar{z})$, Mellin transform $\int_0^\infty d\omega\, \omega^{\Delta-1}$

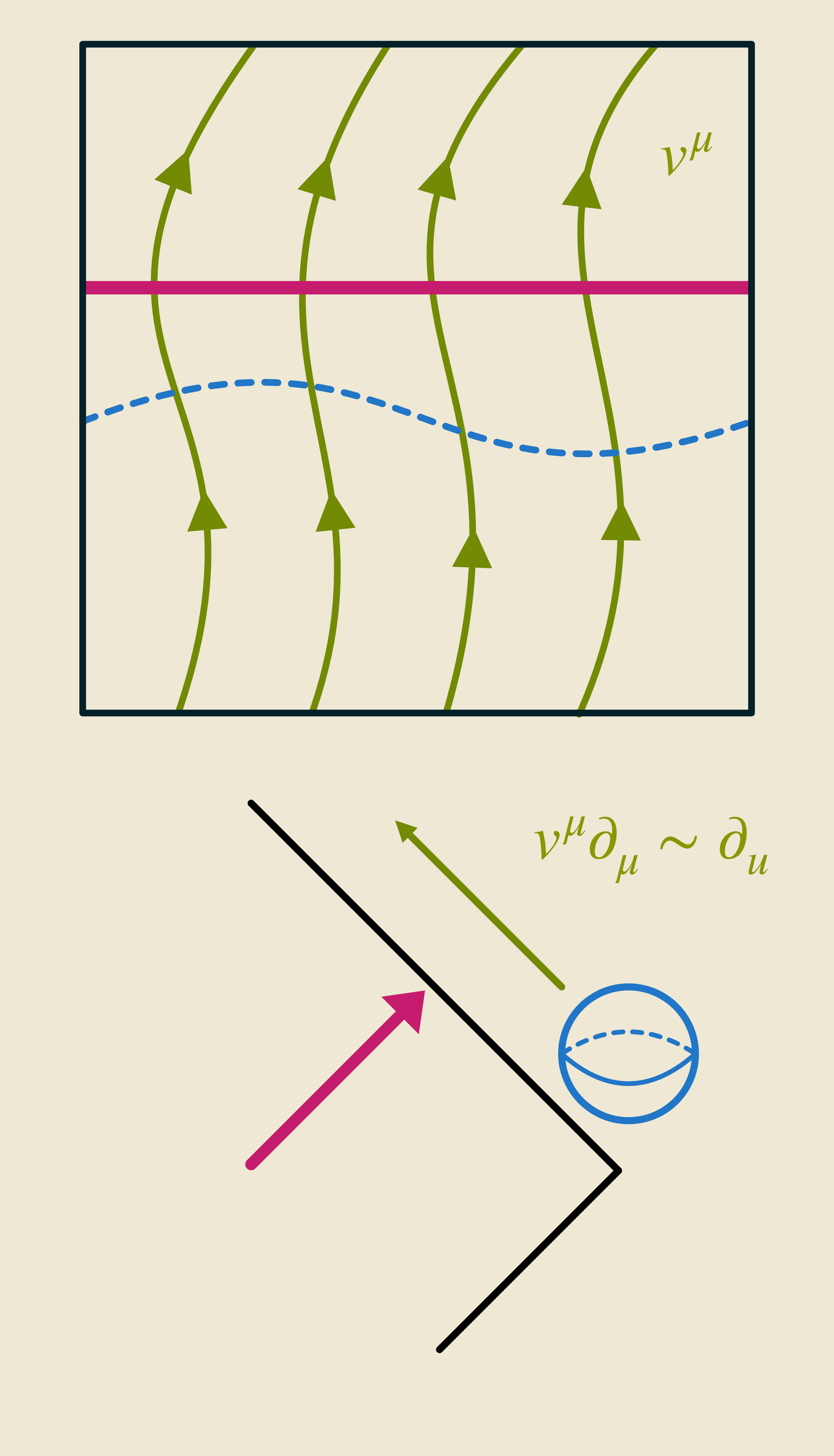
maps $\mathscr{A}(p_i)$ in momentum basis to $\tilde{\mathscr{A}}(\Delta_i, z_i, \bar{z}_i)$ in celestial basis [Pasterski, Shao, Strominger]

But unusual CFT_2 properties! Weight $\Delta \in 1 + i\mathbb{R}$ for basis,
and kinematics restrict two-point ~ $\delta^{(2)}(z,\bar{z})$, three-point vanishing, four-point ~ $\delta^{(2)}(\#)$

- Change signature to $(-+-+)$ eg [Atanasov, Ball, Melton, Raclariu, Strominger]
- Or break supertranslations using background dilaton Φ
  [Fan, Fotopoulos, Stieberger, Taylor, Zhu]

MHV tree-level n-point in Yang-Mills with shock wave profile $\Phi = -\dfrac{1}{2r}\delta(t-r)\,\theta(t)$

reproduces 'regular' 2d *Liouville correlators!* [Stieberger, Taylor, Zhu]



Carroll geometry and flat holography

Holographic dual field theory for asymptotically flat spacetimes?

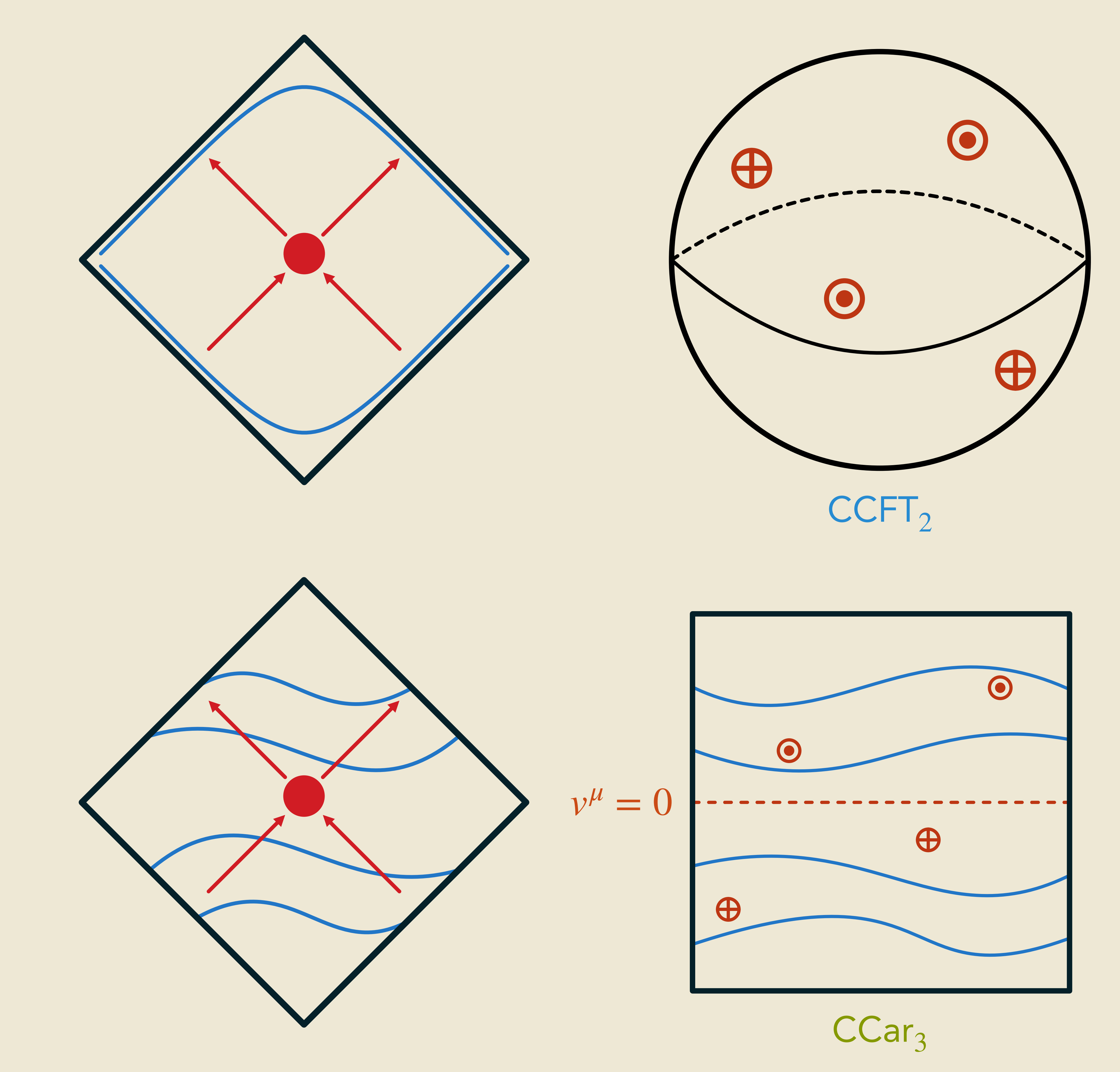
Distinct Cauchy surfaces in bulk:

Celestial CFT_2 ~ S-matrix scattering process

Conformal Carrol_3 ~ natural limit of AdS/CFT?

Related by Fourier and/or (modified) Mellin transform
[Donnay, Fiorucci, Herfray, Ruzziconi] [Bagchi, Banerjee, Basu, Dutta]

But what is overlap?



Carroll geometry and flat holography

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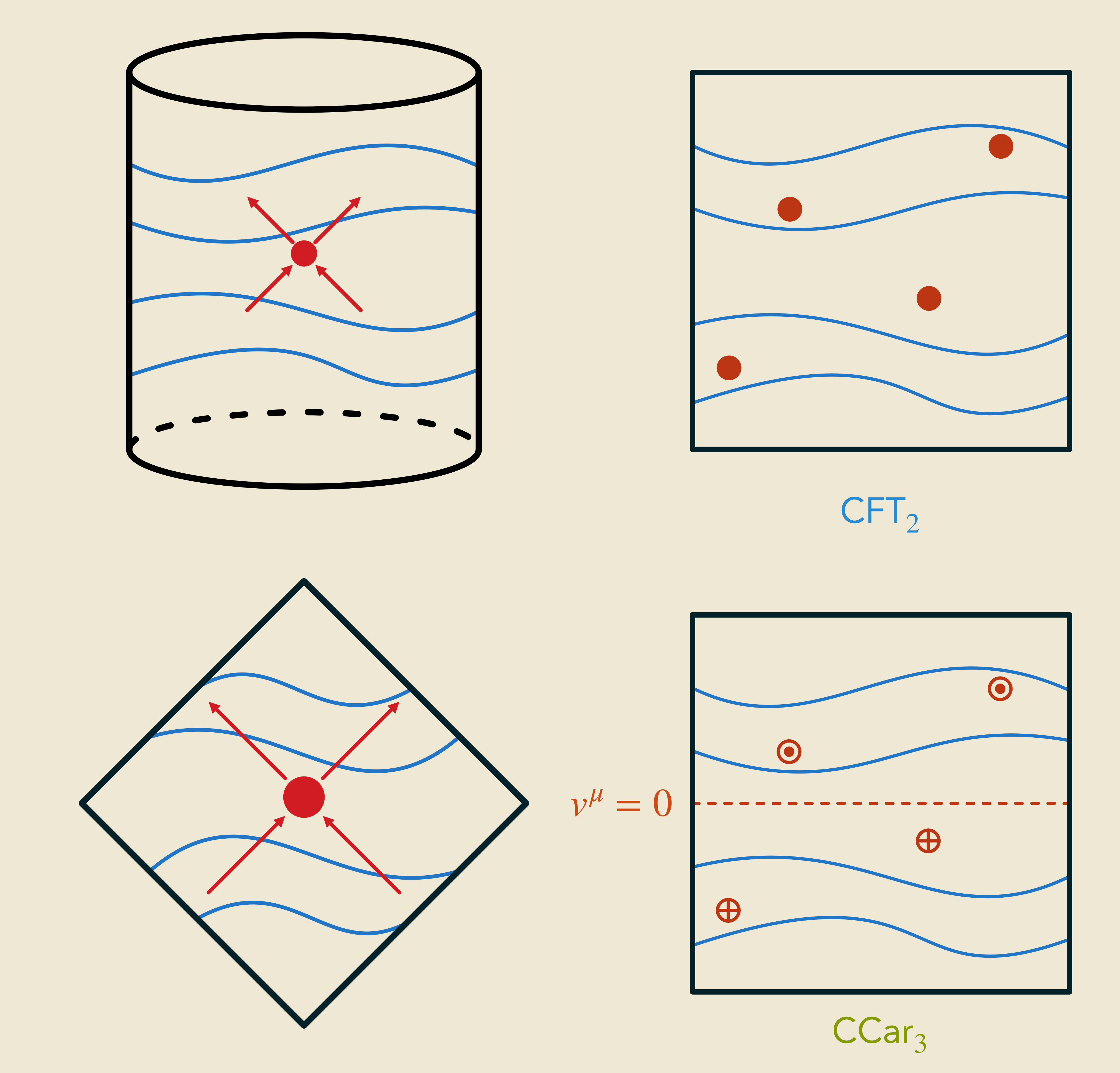
Conformal Carrol_3 from a limit of AdS/CFT?

- Correlators arise from AdS Witten diagrams?
  [Pipolo de Gioia, Raclariu] [Bagchi, Dhivakar, Dutta]
- Need 'leaky' Λ-BMS boundary conditions in AdS to take limit of gravitational asymptotic phase space
  [Compère, Fiorucci, Ruzziconi]
- Hence sources in field theory!
  [Barnich, Fiorucci, Ruzziconi] [Donnay, Herfray, Fiorucci, Ruzziconi]



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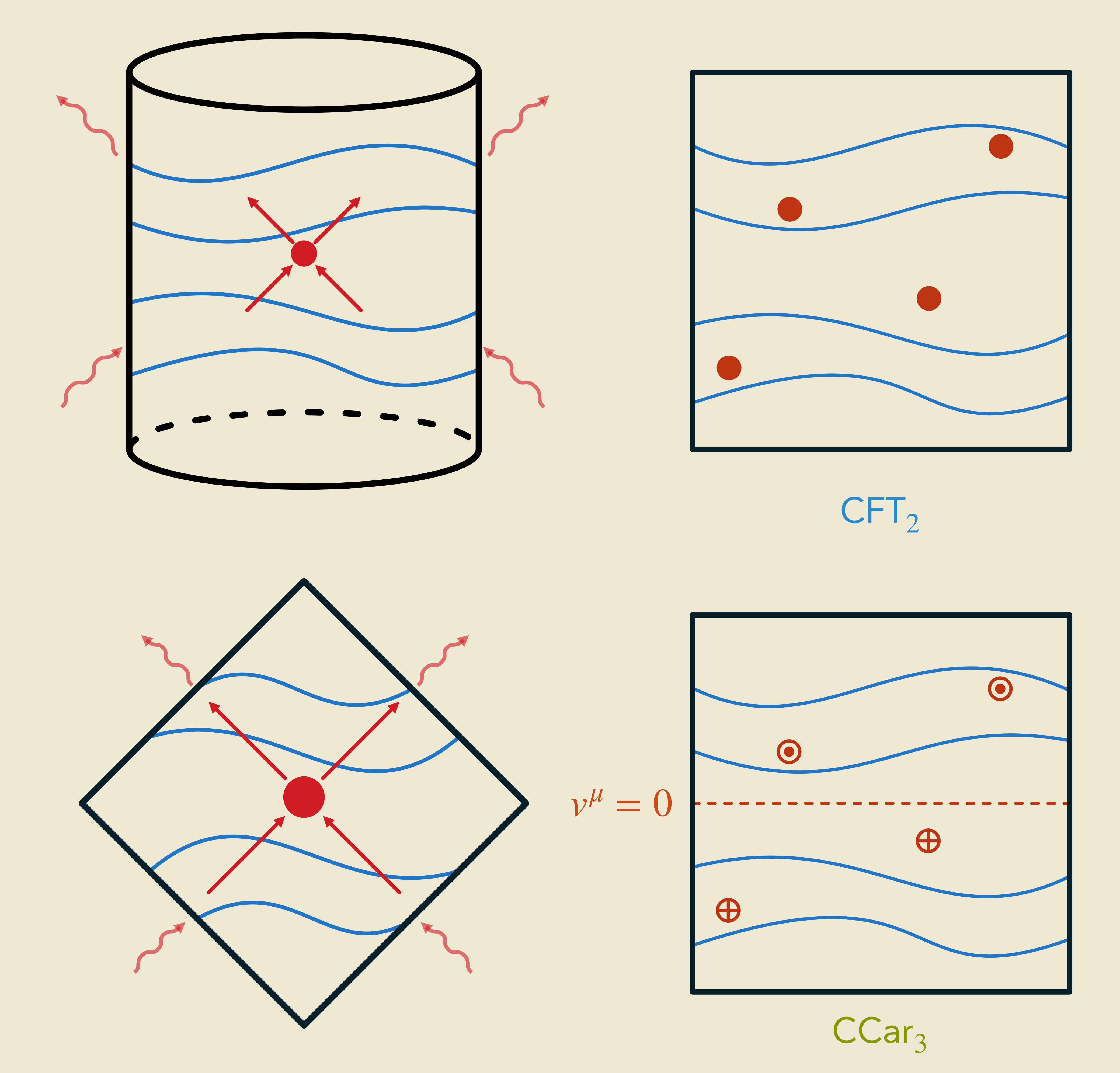

Summary and outlook

Constructed timelike and spacelike conformal Carroll scalar actions

Enables direct computations using only basic QFT techniques

Ongoing and future challenges:

- trace anomalies in conformal Carroll
- further develop conformal Carroll $\Longleftrightarrow$ celestial CFT dictionary
- understand role of 'leakiness'
- explicit actions including fermions and SUSY

Top-down flat holography from $c \to 0$ limit of AdS/CFT?