Carroll and Celestial CFTs

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Based mainly on 2207.03468 with Stefano Baiguera, Watse Sybesma and Benjamin Søgaard

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Carroll symmetries and flat holography

Are used to 'relativistic' Lorentz boosts

$$t \to t + \beta x, \qquad x \to x + \beta t$$

Non-relativistic limit $c \to \infty$ gives Galilean boosts

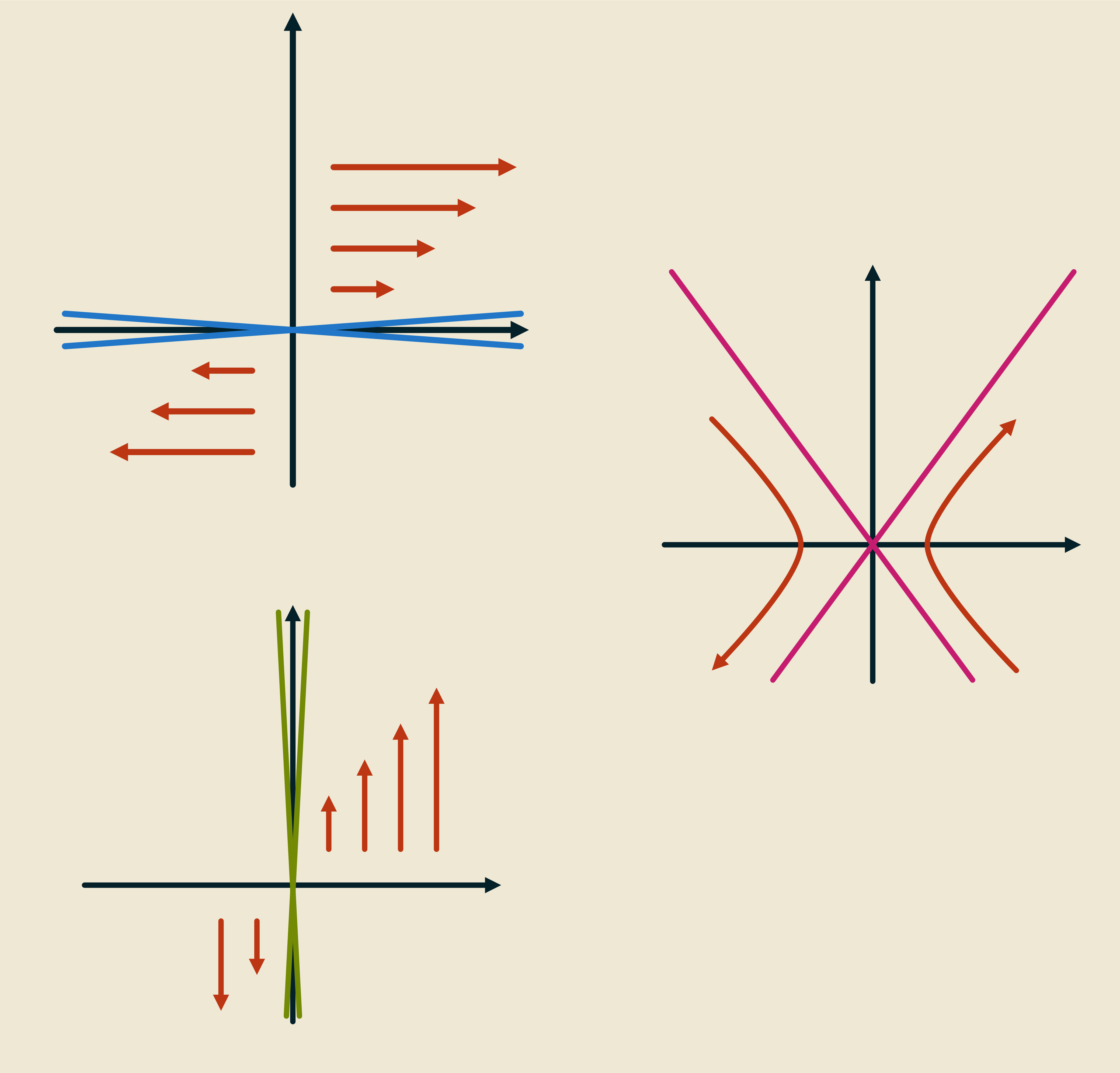
$$t \to t\,, \qquad x \to x + \lambda t$$

Taking $c \to 0$ limit gives Carroll boosts [Levy-Leblond] [Sen Gupta]

$$t \to t + \lambda x, \qquad x \to x$$

Not obviously physical, but:

- ultra-local behavior leads to solvable systems
  integrable BKL-type dynamics in GR [Hansen, Obers, GO, Søgaard]
- BMS = conformal Carroll algebra at $\mathscr{I}^+$
  [Duval, Gibbons, Horvathy, Zhang]
- Flat space holography, relation to celestial approach
  [Donnay, Fiorucci, Herfray, Ruzziconi] [Bagchi, Banerjee, Basu, Dutta]



Carroll symmetries and flat holography

Holographic dual field theory for asymptotically flat spacetimes?

In 3+1 dim: BMS_4 asymptotic symmetries on $\mathscr{I}^+ \simeq \mathbb{R} \times S^2$

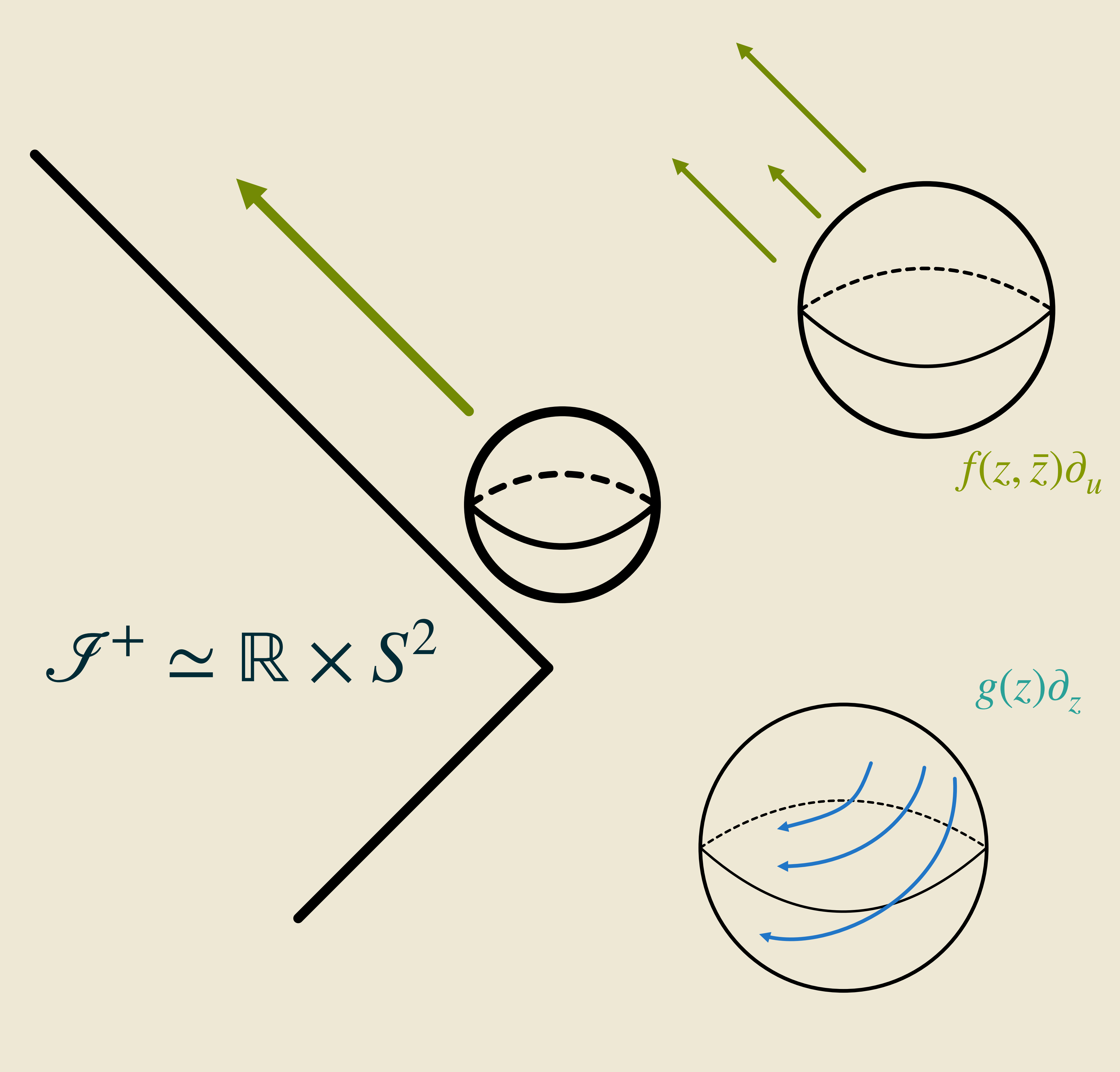
supertranslations $u \to u + f(z, \bar{z})$

- $\sim$ Carroll boosts at each $(z, \bar{z})$
- suggests 3d Carrollian CFT dual: $\mathrm{BMS}_4 \simeq \mathrm{CCar}_3$

superrotations $z \to g(z), \quad \bar{z} \to \bar{g}(\bar{z})$

- Virasoro symmetries of CFT_2
- suggests 2d celestial CFT dual: CCFT_2

Few explicit CCFT_2 theories known,
but *can construct* CCar_3 *examples from* $c \to 0$ *limit!*



Carroll geometry

Are used to 'relativistic' Lorentz boosts

$$t \to t + \beta x, \qquad x \to x + \beta t$$

Taking $c \to 0$ limit gives Carroll boosts [Levy-Leblond] [Sen Gupta]

$$t \to t + \lambda x\,, \quad x \to x \quad \text{and} \quad \partial_t \to \partial_t\,, \quad \partial_x \to \partial_x + \lambda \partial_t$$

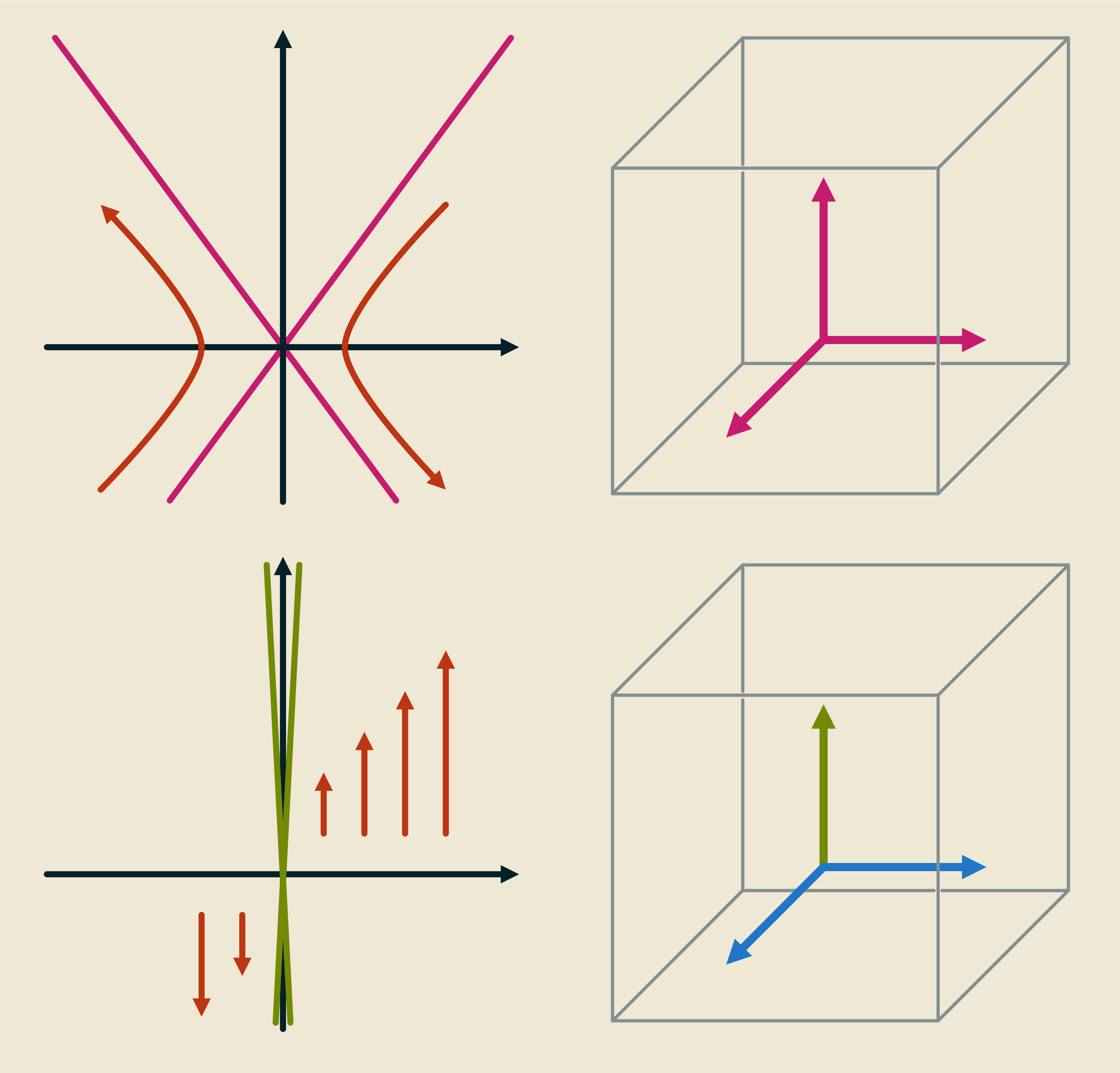
Geometry from spatial metric $h_{\mu\nu}(x^\rho)$ and time vector field $v^\mu(x^\rho)$

Complement with inverse $\tau_\mu(x^\rho)$ and $h^{\mu\nu}(x^\rho)$, satisfy

$$v^\mu h_{\mu\nu} = 0, \quad \tau_\mu h^{\mu\nu} = 0, \quad v^\mu \tau_\mu = -1, \quad \delta^\mu_\nu = -v^\mu \tau_\nu + h^{\mu\rho} h_{\rho\nu}$$

Transform under local Carroll boosts $\lambda_\mu(x^\rho)$ as

$$\delta_\lambda \tau_\mu = \lambda_\mu\,, \qquad \delta_\lambda h^{\mu\nu} = \lambda^\mu v^\nu + v^\mu \lambda^\nu$$



[Duval, Gibbons, Horvathy, Zhang] [Hartong] [Ciambelli, Marteau, Petropoulos…]
[Hansen, Obers, GO, Søgaard] …

Conformal scalar actions: timelike

Consider Lorentzian conformal scalar action,

$$S = -\frac{1}{2}\int d^d x\sqrt{-g}\left(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{d-2}{4(d-1)}R\phi^2\right)$$

In Carroll limit $c \to 0$, leading-order terms give [Baiguera, GO, Sybesma, Søgaard]

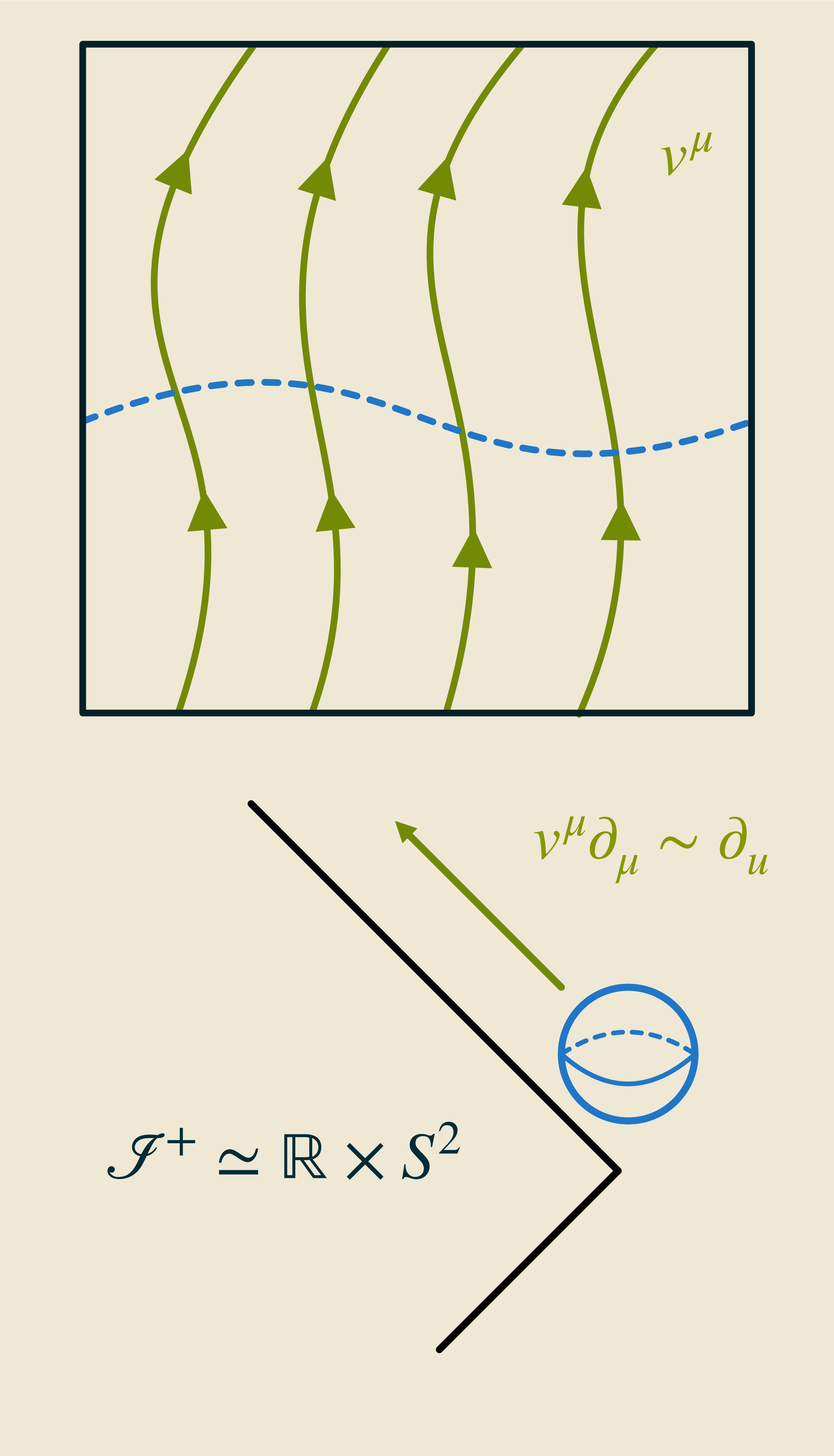
$$S_t = -\frac{1}{2}\int d^d x\, e\left[-(v^\mu\partial_\mu\phi)^2 + \frac{(d-2)}{4(d-1)}\left(K^{\mu\nu}K_{\mu\nu} + K^2 - 2v^\mu\partial_\mu K\right)\phi^2\right]$$

where $K_{\mu\nu} = -\frac{1}{2}\mathscr{L}_v h_{\mu\nu}$ is extrinsic curvature

This is timelike conformal Carroll scalar, flat space propagator $\sim u\,\delta^{(2)}(z,\bar{z})$

Carroll boost-invariant and Weyl-invariant, so ${T^i}_0 = 0$ and ${T^\mu}_\mu = 0$

Also considered from no-boost approach in [Gupta, Suryanarayana] [Rivera-Betancour, Vilatte]



Conformal scalar actions: spacelike

Consider Lorentzian conformal scalar action,

$$S = -\frac{1}{2}\int d^d x \sqrt{-g}\left(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{d-2}{4(d-1)}R\phi^2\right)$$

Alternative Carroll limit $c \to 0$ together with two constraints gives

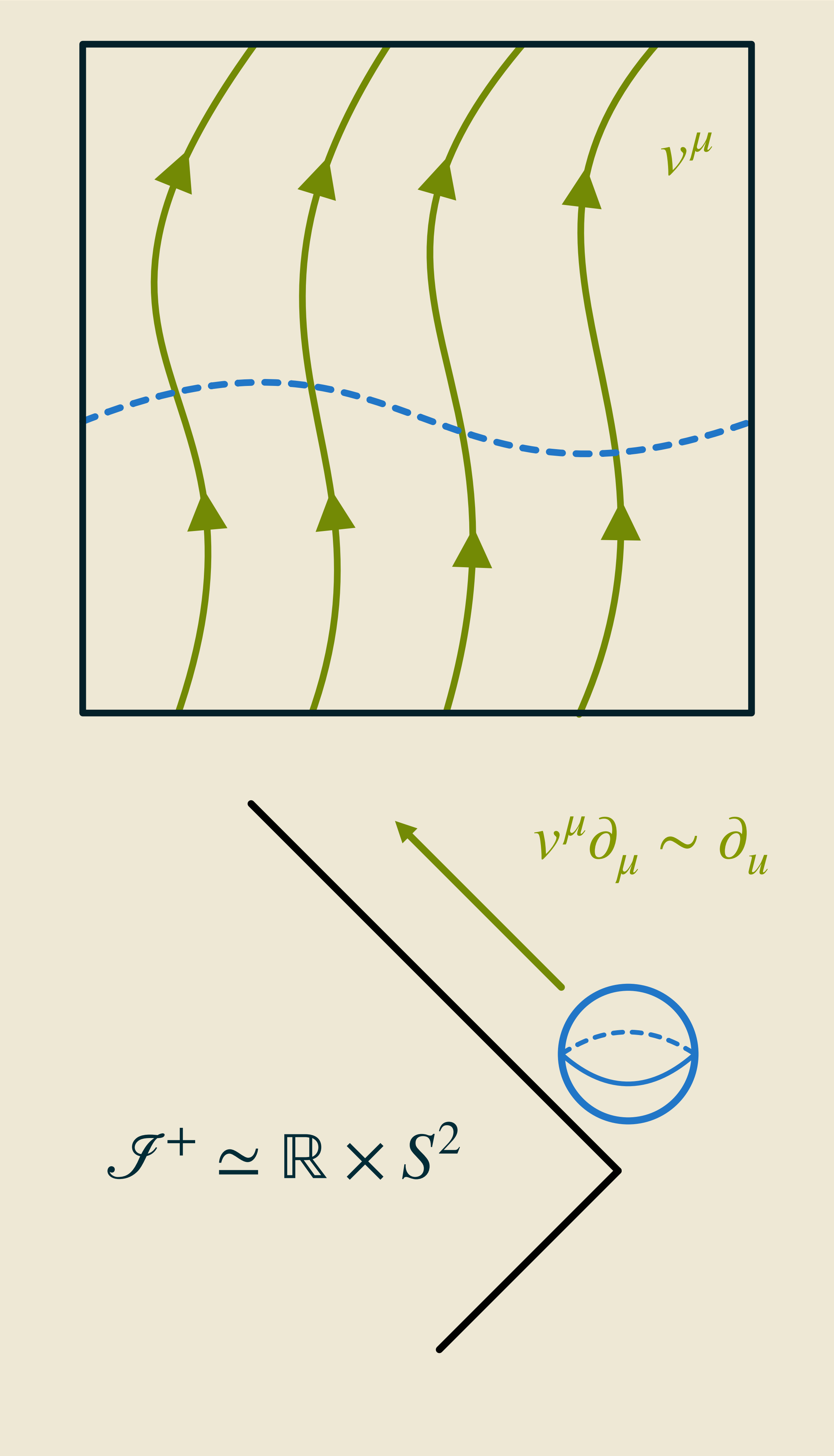
$$S_s = -\frac{1}{2}\int d^d x\, e\left[h^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{(d-2)}{4(d-1)}\left(h^{\mu\nu}\tilde{R}_{\mu\nu} - \tilde{\nabla}_\mu a^\mu\right)\right]$$

with

- time-dependence fixed by $v^\mu\partial_\mu\phi = -\frac{(d-2)}{4(d-1)}K$
- extrinsic curvature must be pure trace $K_{\mu\nu} = \frac{h_{\mu\nu}}{d-1}K$

This is spacelike conformal Carroll scalar. [Baiguera, GO, Sybesma, Søgaard]

Boost- and Weyl-invariant, flat space propagator $\sim \log(x)^2$ spacelike



Conformal scalar actions: spacelike

Can dimensionally reduce spacelike action

$$S_s \implies -\frac{1}{2}\int d^{d-1}x\sqrt{h}\,\left(h^{\mu\nu}\partial_\mu\hat{\phi}\partial_\nu\hat{\phi} + \frac{(d-3)}{4(d-2)}\hat{R}\hat{\phi}^2 + \frac{1}{4(d-1)(d-2)}A^{-2}\hat{R}_{A^{-2}h_{ij}}\right)$$

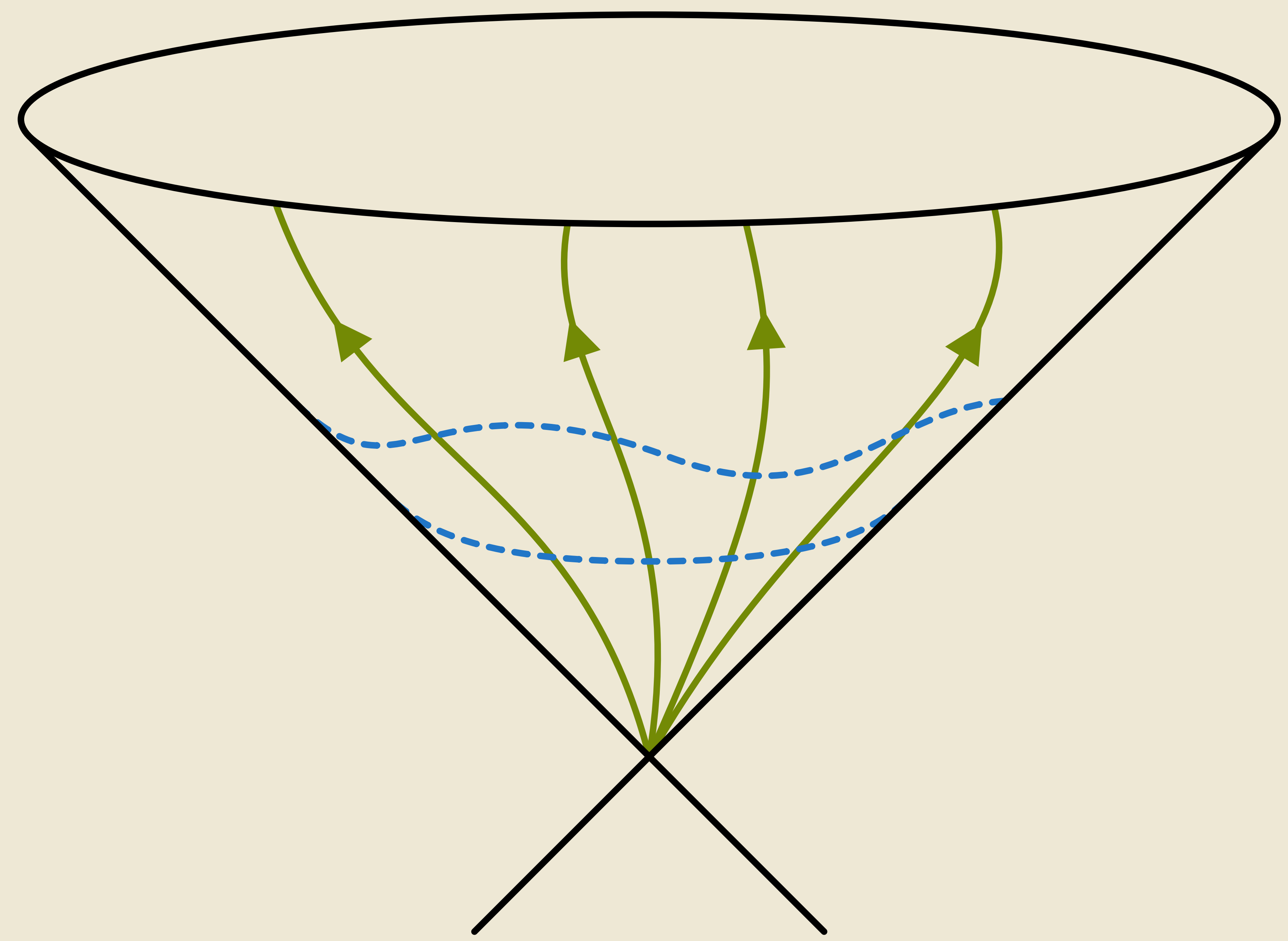
Reminiscent of embedding space formalism!

Get $(d-1)$-dim conformal $SO(d,1)$ representations
from $(d+1)$-dim Lorentz representations in $\mathbb{R}^{1,d}$

Restriction to light cone
$\implies$ Carrollian spacelike theory
$\implies$ Euclidean theory

Similar procedure for other spacelike Carroll theories?

Also `regular' CFT applications? [Parisini, Skenderis, Withers]



To boost or not to boost?

Local Carroll boost symmetry

- inevitable for limit of Lorentz-invariant theory
- implies vanishing energy flux ${T^i}_0 = 0$
- 'timelike' or 'spacelike' $\langle \phi(u,z,\bar{z})\, \phi(0,0,0) \rangle = \begin{cases} f(u)\delta^{(2)}(z,\bar{z}) \\ g(z) \end{cases}$

[Henneaux, Salgado-Rebodello] [De Boer, Hartong, Obers, Sybesma, Vandoren]

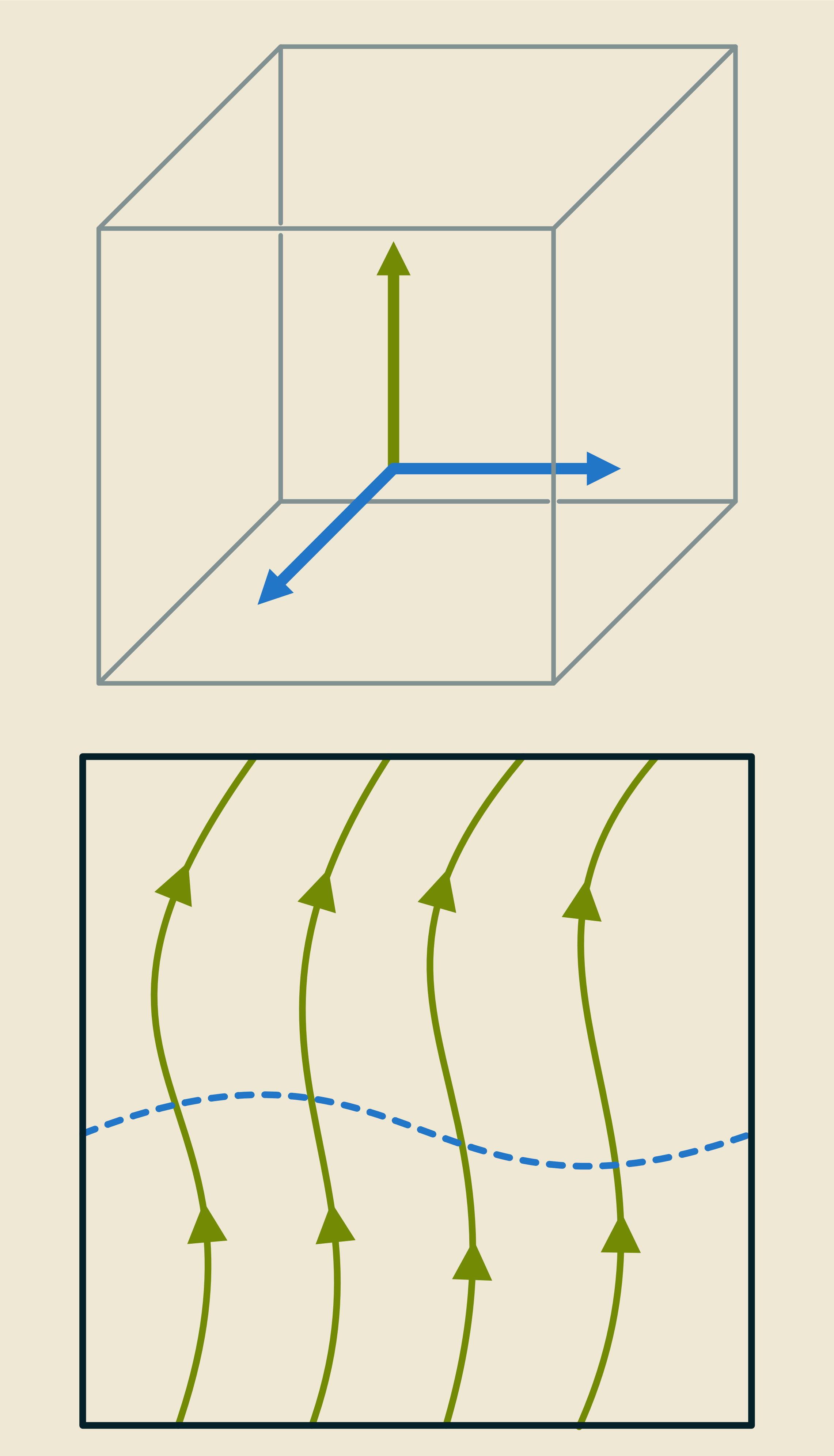
Timelike branch reproduces CCFT correlators [Bagchi, Banerjee, Basu, Dutta]

However maybe *Carroll boosts not always desired* in flat space holography?

- known holographic fluids with ${T^i}_0 \neq 0$ [Ciambelli, Marteau, Petkou, Petropoulos, Siampos]
- focus instead on $(v^\mu, h_{\mu\nu})$ fiber structure?

  [Ciambelli, Leigh, Marteau, Siampos] [Petkou, Petropoulos, Rivera Betancour, Siampos] [Freidel, Jai-akson]…
- go to Lorentz-breaking frame before taking flat/Carroll limit in AdS/CFT?

  cf [Campoleoni, Ciambelli, Delfante, Marteau, Petropoulos, Ruzziconi]



To boost or not to boost?

Breaking Carroll boost ~ breaking supertranslation symmetry in celestial holography

For massless particles with $p^\mu = \omega\, q^\mu(z,\bar{z})$, Mellin transform $\int_0^\infty d\omega\, \omega^{\Delta-1}$

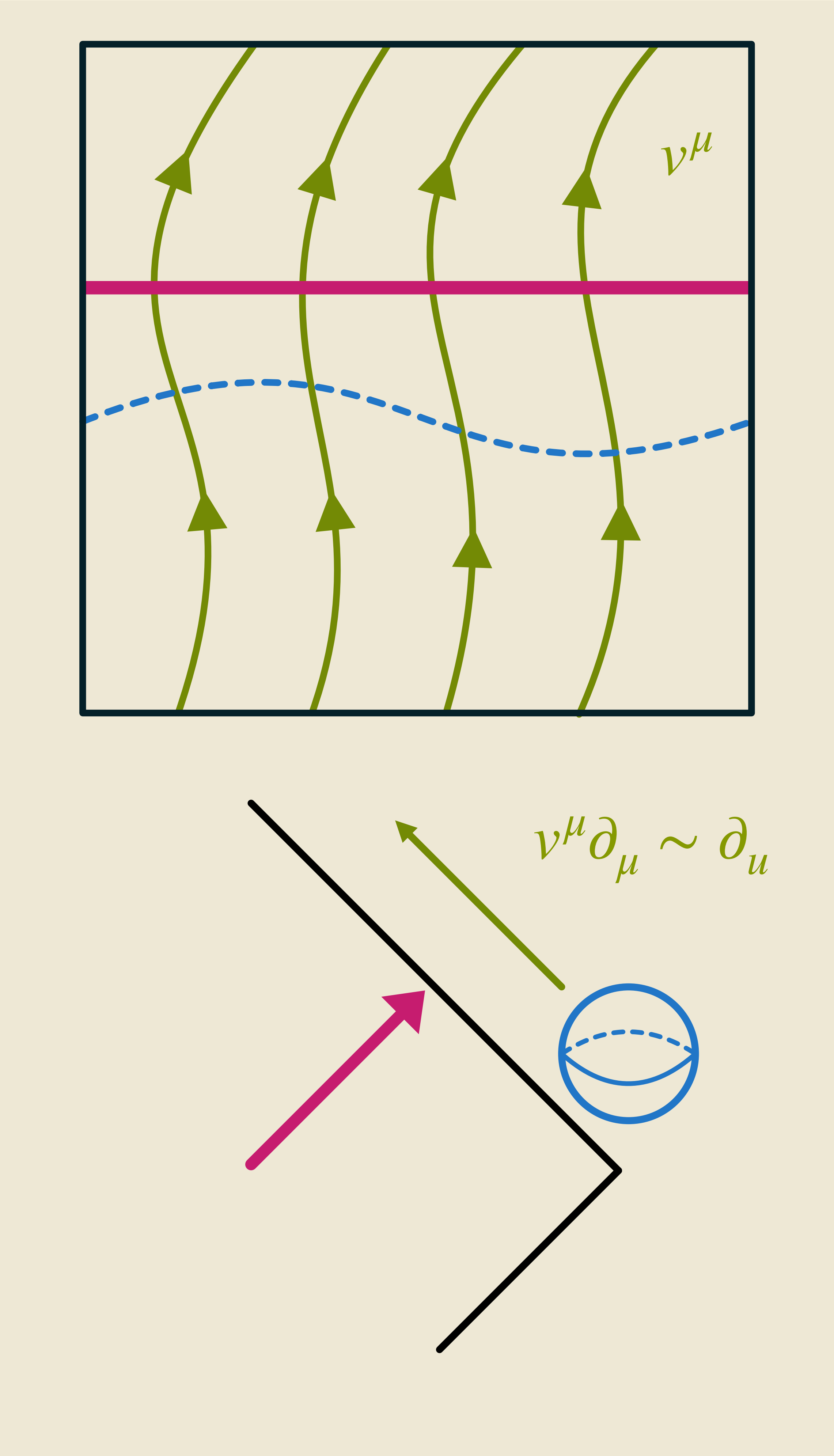
maps $\mathscr{A}(p_i)$ in momentum basis to $\tilde{\mathscr{A}}(\Delta_i, z_i, \bar{z}_i)$ in celestial basis [Pasterski, Shao, Strominger]

But unusual CFT_2 properties! Weight $\Delta \in 1 + i\mathbb{R}$ for basis,
and kinematics restrict two-point ~ $\delta^{(2)}(z,\bar{z})$, three-point vanishing, four-point ~ $\delta^{(2)}(\#)$

- Change signature to $(-+-+)$ eg [Atanasov, Ball, Melton, Raclariu, Strominger]
- Or break supertranslations using background dilaton Φ [Fan, Fotopoulos, Stieberger, Taylor, Zhu]

MHV tree-level n-point in Yang-Mills with shock wave profile $\Phi = -\frac{1}{2r}\delta(t-r)\,\theta(t)$

reproduces 'regular' 2d *Liouville correlators!* [Stieberger, Taylor, Zhu]



Summary and outlook

Constructed timelike and spacelike conformal Carroll scalar actions

Allow explicit computations using only basic QFT techniques

Ongoing and future work:

- study sources and breaking of boosts ~ supertranslations
- relation to `spatial' Liouville amplitudes from YM + dilaton?
- conformal Carroll trace anomalies, classification?

Build up conformal Carroll $\Longleftrightarrow$ CCFT dictionary

Top-down flat holography from $c \to 0$ limit of AdS/CFT?