# Some* aspects of non-relativistic strings

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* and definitely not *all* aspects — see also review paper 2202.12698 with Ziqi Yan

# Outline

- Introduction: Gomis-Ooguri limit
- Warmup: non-relativistic point particle
- Gomis-Ooguri strings in curved backgrounds
- Spin Matrix limits of strings on AdS
- Outlook

# Gomis-Ooguri limit

Start from relativistic strings in flat background with compact $X^1 \sim X^1 + 2\pi R$

$$S = \frac{1}{4\pi\hat{\alpha}'}\int d^2\sigma \left(\partial_\alpha X^\mu \partial^\alpha X^\nu \hat{G}_{\mu\nu} - i\epsilon^{\alpha\beta}\partial_\alpha X^\mu \partial^\beta X^\nu \hat{B}_{\mu\nu}\right)$$

Distinguish longitudinal $X^A = (X^0, X^1)$ and transverse $X^i = (X^2, \ldots, X^d)$ directions

$$\hat{G}_{\mu\nu} = \begin{pmatrix} \eta_{AB} & 0 \\ 0 & \frac{\hat{\alpha}'}{\alpha'}\delta_{ij} \end{pmatrix}, \qquad \hat{B}_{\mu\nu} = \begin{pmatrix} -\epsilon_{AB} & 0 \\ 0 & 0 \end{pmatrix}$$

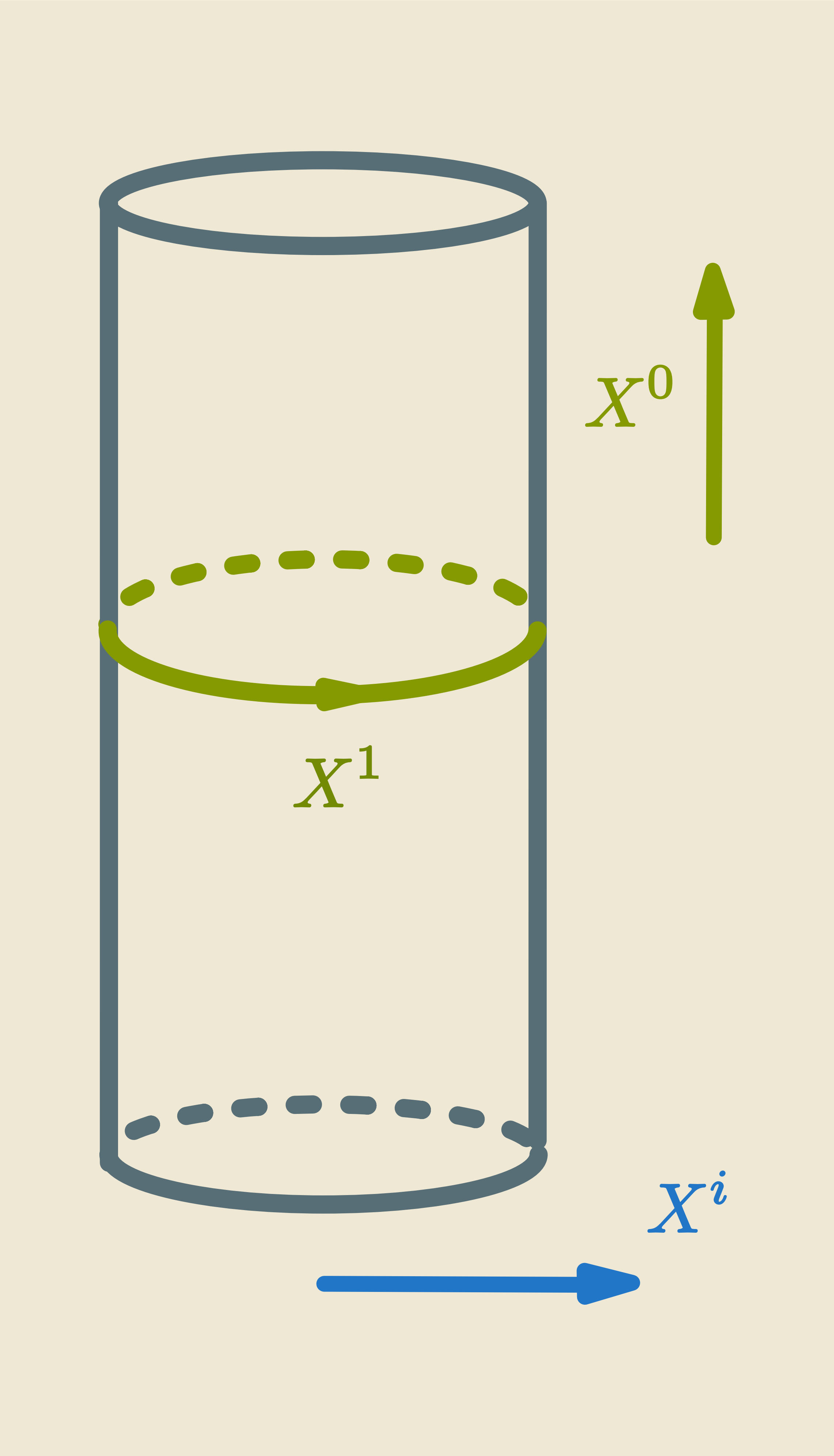
Spectrum with $X^1$ momentum $n$ and winding $w$

$$\left(E + \frac{wR}{\hat{\alpha}'}\right)^2 - \frac{\alpha'}{\hat{\alpha}'}p^i p_i = \frac{n^2}{R^2} + \frac{w^2R^2}{\hat{\alpha}'^2} + \frac{2}{\hat{\alpha}'}(N + \bar{N} - 2)$$

In limit $\hat{\alpha}' \to 0$ get *non-relativistic spectrum* for $w \neq 0$ ,

$$E = \frac{\alpha'}{2wR}\left[p^i p_i + \frac{2}{\alpha'}(N + \bar{N} - 2)\right]$$

[Gomis, Ooguri] [Danielsson, Guijosa, Kruczenski]



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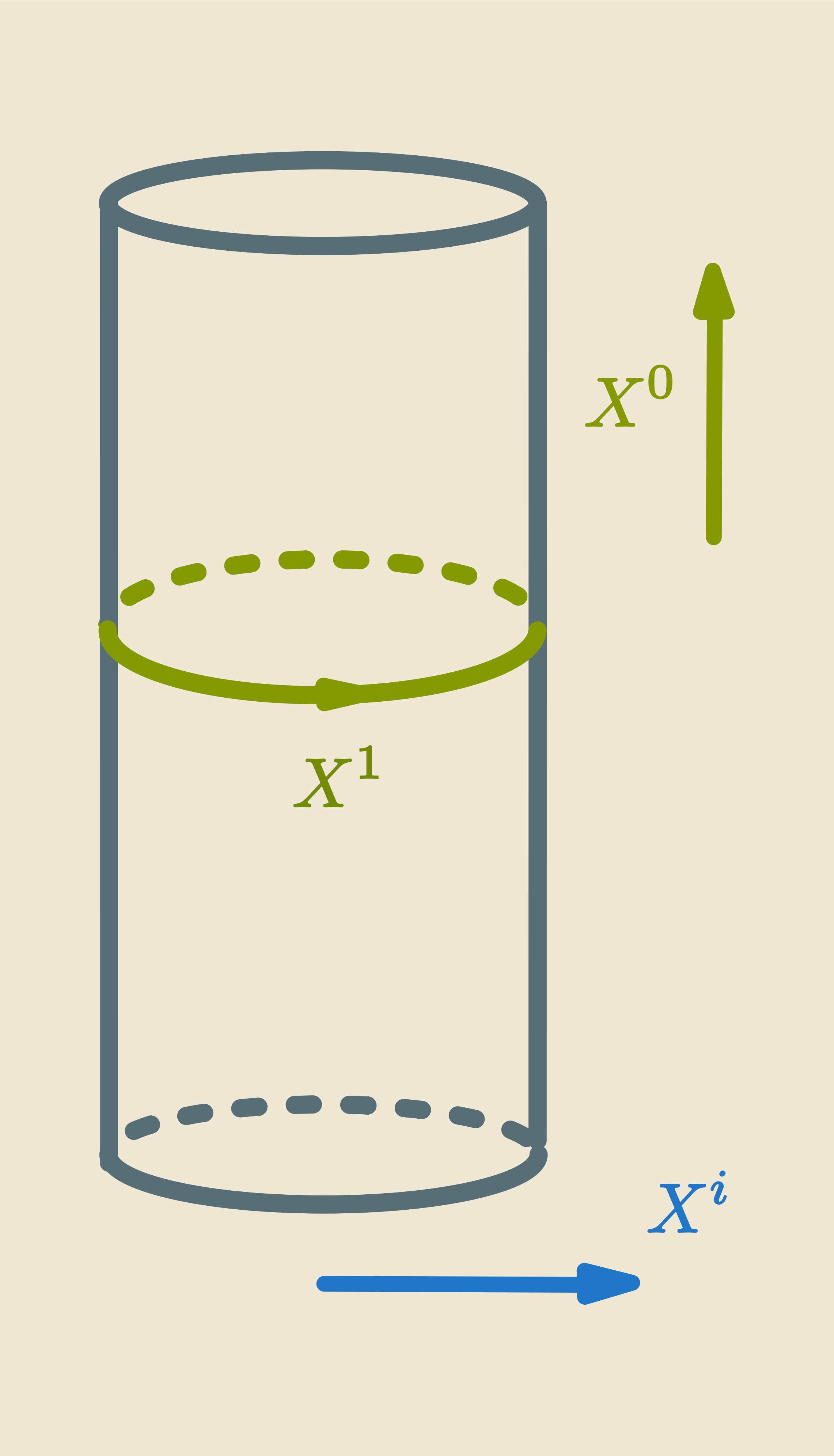
For action? Rewrite using Lagrange multipliers $\lambda$ and $\bar{\lambda}$ ,

$$S = \frac{1}{4\pi\hat{\alpha}'}\int d^2\sigma \left(\partial_\alpha X^i \partial^\alpha X_i + \lambda\bar{\partial}X + \bar{\lambda}\partial\bar{X} + \frac{\hat{\alpha}'}{\alpha'}\lambda\bar{\lambda}\right)$$

In non-relativistic limit $\hat{\alpha}' \to 0$ get *Gomis-Ooguri action* [Gomis, Ooguri]

$$S = \frac{1}{4\pi\alpha'}\int d^2\sigma \left(\partial_\alpha X^i \partial^\alpha X_i + \lambda\bar{\partial}X + \bar{\lambda}\partial\bar{X}\right)$$



# Gomis-Ooguri limit

Gomis-Ooguri string with non-relativistic spectrum

$$S = \frac{1}{4\pi\alpha'} \int d^2\sigma \left( \partial_\alpha X^i \partial^\alpha X_i + \lambda \bar{\partial} X + \bar{\lambda} \partial \bar{X} \right)$$

Motivated by non-commutative open string (NCOS) limits
[Gomis, Ooguri] [Danielsson, Guijosa, Kruczenski]

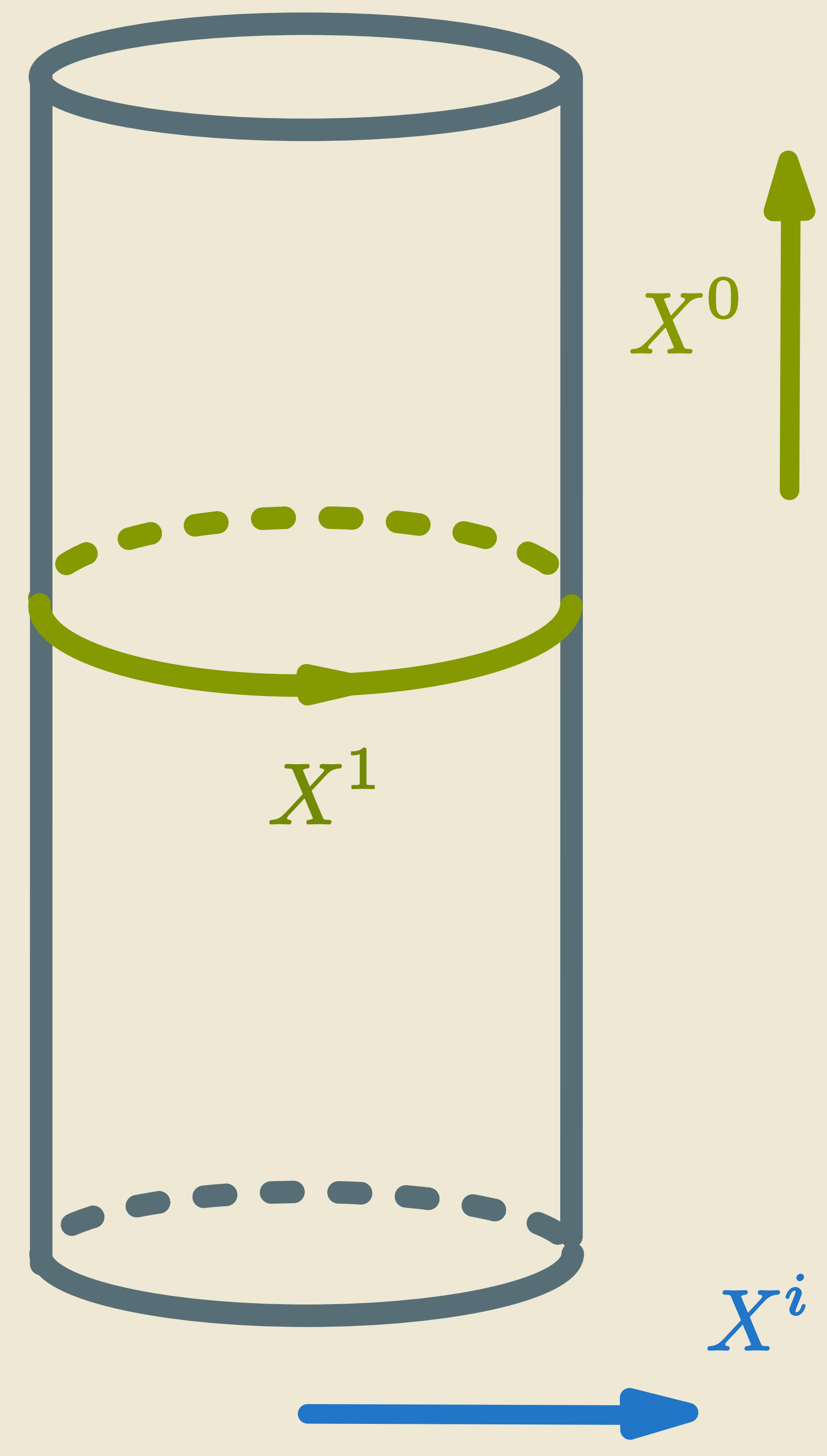
- Lorentzian $CFT_2$ on worldsheet
- UV-complete theory
- Simple moduli space at one loop
- Interacting worldsheet: flow back to relativistic if $\lambda\bar{\lambda}$ coupling is turned on! [Yan]

T-duality along compact spatial $X^1 \sim X^1 + 2\pi R$ direction gives

$$S = \frac{1}{4\pi\alpha'} \int d^2\sigma \left( \partial_\alpha X^i \partial^\alpha X_i - 2\partial Y^1 \bar{\partial} X^0 - 2\bar{\partial} Y^1 \partial X^0 \right)$$

Dual $Y^1$ direction is null and compact: $Y^1 \sim Y^1 + 2\pi\alpha'/R \implies$ DLCQ of string theory!

What happens with target space geometry in limit? [Andringa, Bergshoeff, Gomis, De Roo]



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# Reminder: non-relativistic particle

Lorentzian point particle action

$$S = mc \int d\lambda \sqrt{-g_{\mu\nu} \dot{X}^\mu(\lambda) \dot{X}^\nu(\lambda)} + q \int d\lambda \, A_\mu \dot{X}^\mu$$

On flat background, with time $X^0$ and space $X^i$,

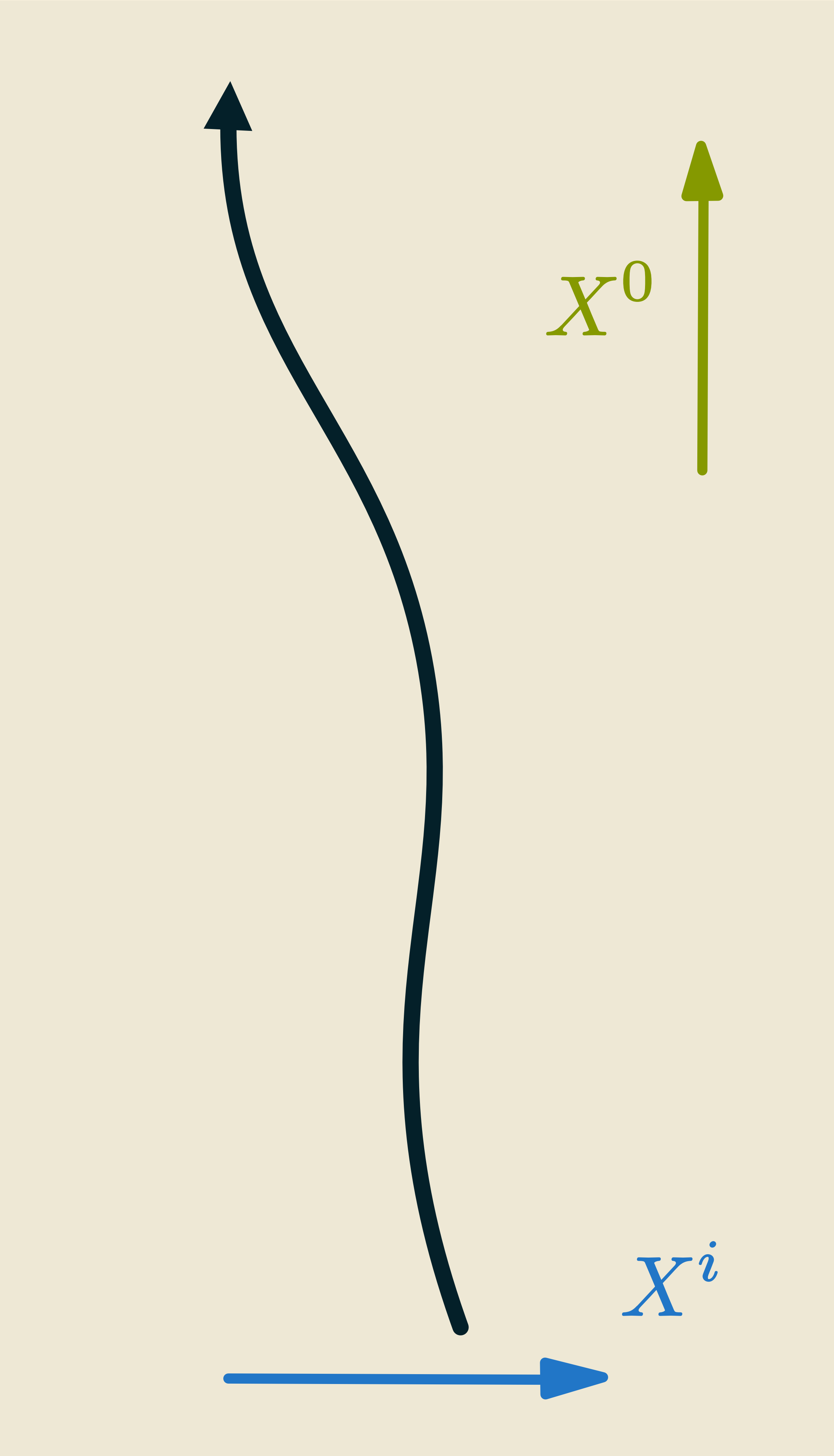
$$S = -mc \int d\lambda \sqrt{c^2 \left(\dot{X}^0\right)^2 - \delta_{ij} \dot{X}^i \dot{X}^j}$$

$$= -mc^2 \int d\lambda \, \dot{X}^0 + \frac{m}{2} \int d\lambda \frac{\delta_{ij} \dot{X}^i \dot{X}^j}{\dot{X}^0} + \cdots$$

Get divergence from rest mass as $c \to \infty$, cancel using electric coupling $qA_0 = mc^2$

$$S = \frac{m}{2} \int d\lambda \, \frac{\delta_{ij} \dot{X}^i \dot{X}^j}{\dot{X}^0}$$

Usual non-relativistic particle action, can gauge fix $X^0(\lambda) = \lambda$



# Reminder: non-relativistic particle

Symmetries of non-relativistic particle action?

$$S = \frac{m}{2}\int d\lambda\, \frac{\delta_{ij}\dot{X}^i\dot{X}^j}{\dot{X}^0}$$

Galilean boosts $X^i \to X^i + v^i X^0$ and translations $X^i \to X^i + w^i$ give $\{Q^G, Q^P\} = -\, m\, v\cdot w$

$\Longrightarrow$ central *Bargmann extension*

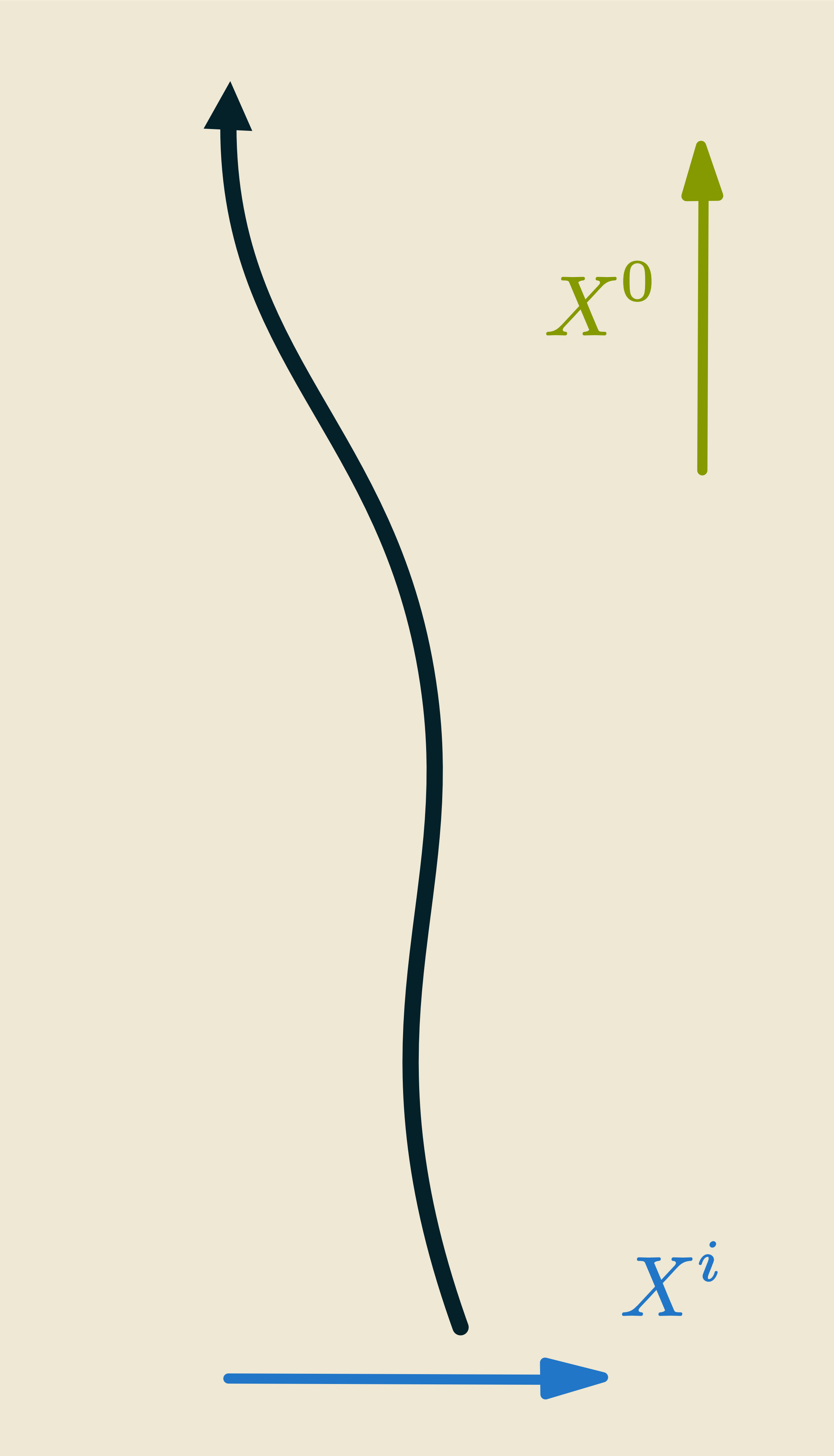
Extra background field? Decompose Lorentzian metric and electromagnetic field as

$$g_{\mu\nu} = -\,c^2\tau_\mu\tau_\nu + h_{\mu\nu} - \tau_\mu m_\nu - m_\mu\tau_\nu + \cdots$$
$$qA_\mu = mc^2\tau_\mu + qa_\mu + \cdots$$

then the limit of the action gives, denoting $\bar{h}_{\mu\nu} = h_{\mu\nu} - \tau_\mu m_\mu - m_\mu\tau_\nu$

$$S = \frac{m}{2}\int d\lambda\, \frac{\bar{h}_{\mu\nu}\dot{X}^\mu\dot{X}^\nu}{\tau_\rho\dot{X}^\rho} + q\int d\lambda\, a_\mu\dot{X}^\mu$$

Couples to *Bargmann field* $m_\mu$ from subleading `time' metric



# Reminder: non-relativistic particle

What happens on the level of symmetry algebra?

Poincaré symmetry of flat Lorentzian geometry plus $U(1)$ gauge field

$$\mathscr{A}_\mu = E_\mu{}^A P_A + \frac{1}{2}\Omega_\mu{}^{AB} M_{AB} + A_\mu Q$$

Redefine $E_\mu{}^A = \left(c\tau_\mu + \frac{1}{c}m_\mu\,, e_\mu{}^a\right)$ and $A_\mu = \tau_\mu$ so with $H = cP_0 + Q$ and $N = \frac{1}{c}P_0$ get

$$\mathscr{A}_\mu = \tau_\mu H + e_\mu{}^a P_a + m_\mu N + \frac{1}{2}\Omega_\mu{}^{ab} J_{ab} + \Omega_\mu{}^a G_a$$

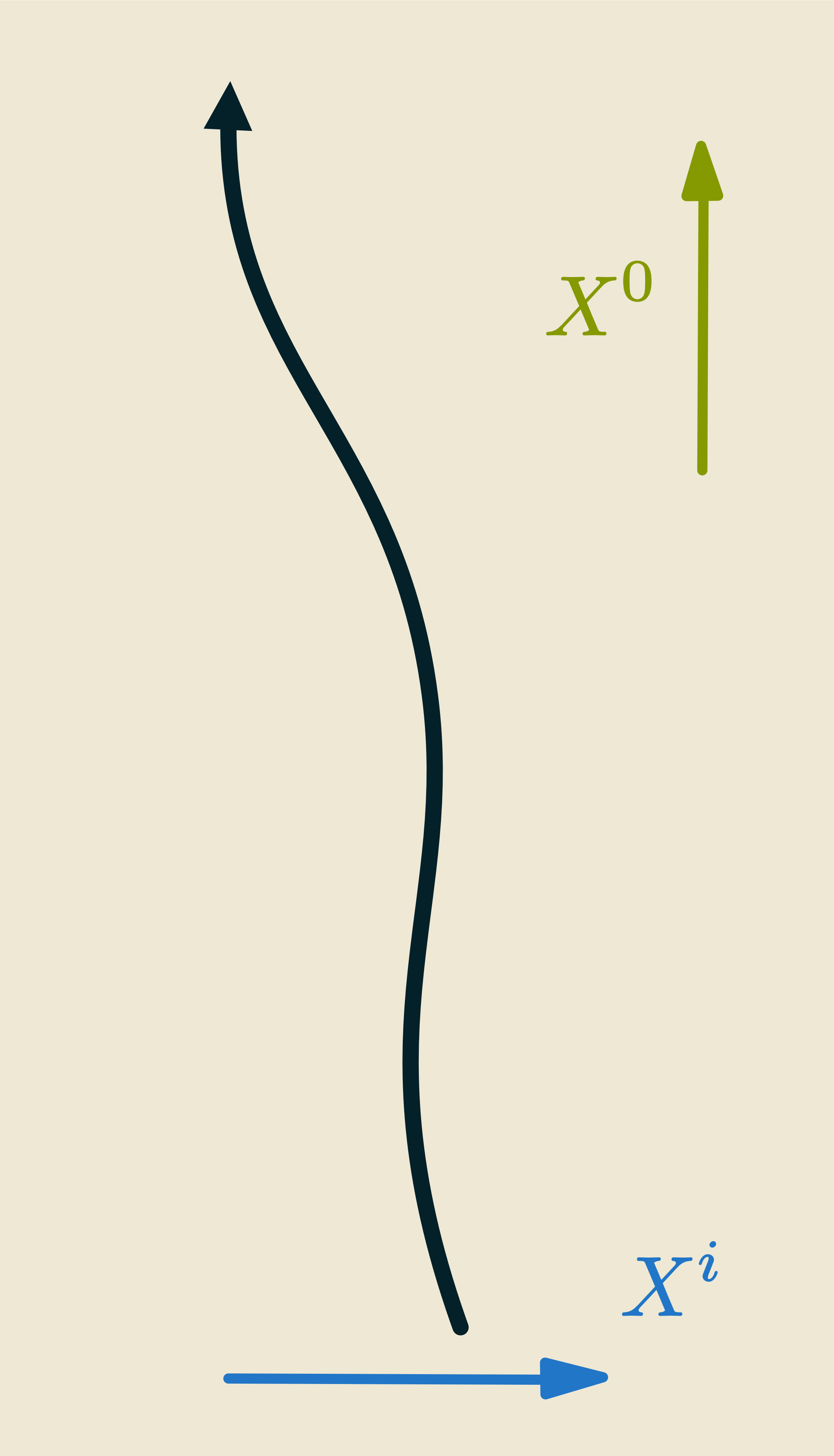
İnönü-Wigner contraction $c \to \infty$ gives Bargmann algebra

$$[J_{ab}, J_{cd}] = \delta_{ac}J_{bd} - \delta_{bc}J_{ad} + \delta_{bd}J_{ac} - \delta_{ad}J_{bc}\,,$$

$$[J_{ab}, P_c] = \delta_{ac}P_b - \delta_{bc}P_a\,, \qquad [J_{ab}, G_c] = \delta_{ac}G_b - \delta_{bc}G_a\,,$$

$$[G_a, H] = -P_a\,, \qquad [G_a, P_b] = -\delta_{ab}N\,.$$

Galilean boosts $G_a$ act as $\delta_\lambda h_{\mu\nu} = \tau_\mu\lambda_\mu + \lambda_\mu\tau_\nu$ and $\delta_\lambda m_\mu = \lambda_\mu$ , leave $\bar{h}_{\mu\nu}$ invariant



# Reminder: non-relativistic particle

Can get same non-relativistic point particle action from null reduction on background

$$ds^2 = 2\tau_\mu dx^\mu \left(du - m_\nu dx^\nu\right) + h_{\mu\nu} dx^\mu dx^\nu$$

from massless particle with $p_u = m$ , Bargmann algebra now arises from centralizer of $P_u$

Both give 'type I' torsional Newton-Cartan geometry (TNC), no constraints on torsion $d\tau$

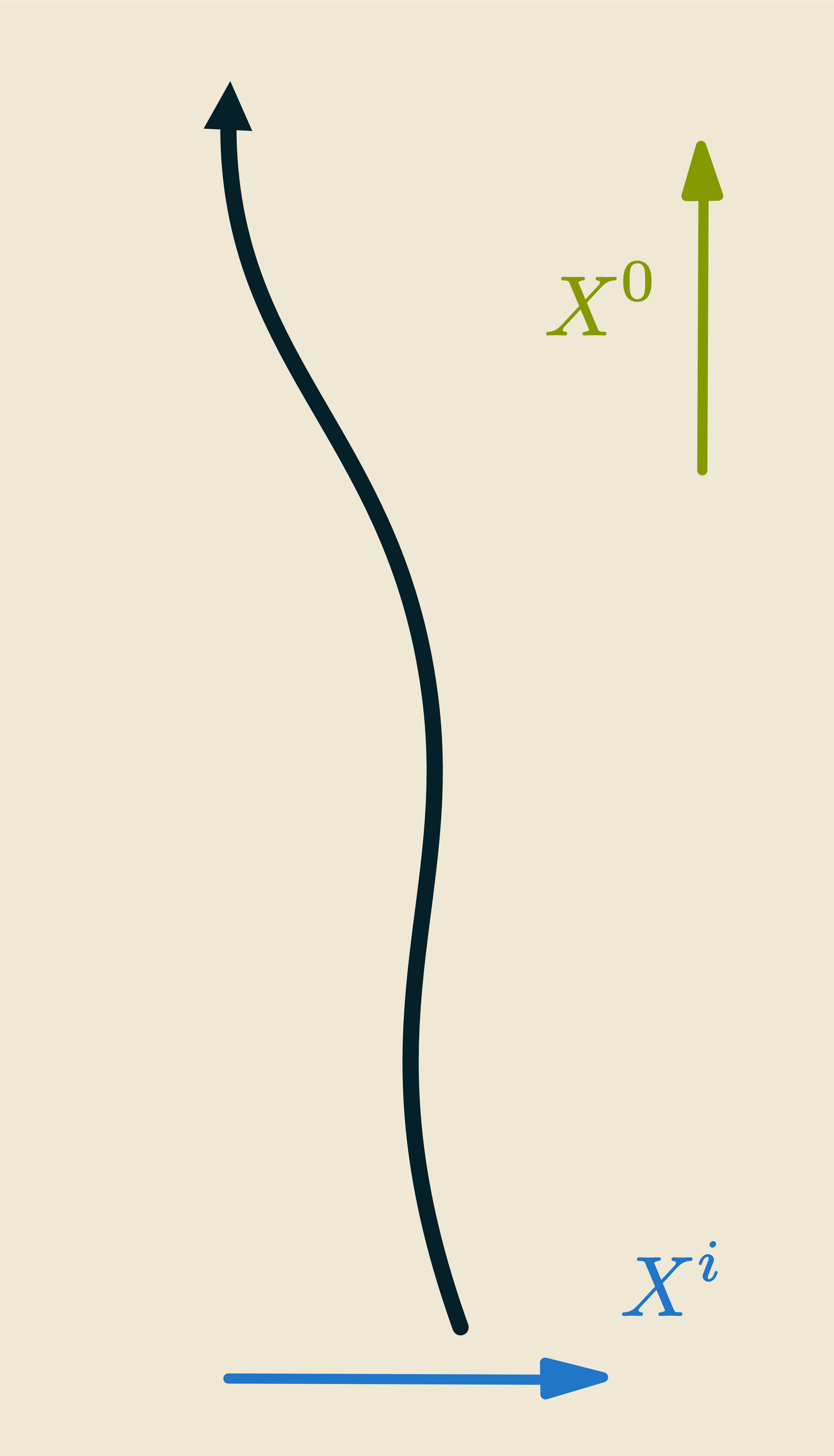
Note 'Stückelberg' symmetry between Bargmann $m_\mu$ and electromagnetic $a_\mu$ fields,

$$S = \frac{m}{2}\int d\lambda \frac{\bar{h}_{\mu\nu} X^\mu(\lambda) X^\nu(\lambda)}{(\tau_\rho X^\rho(\lambda))^2} + \int d\lambda \left(m\, m_\mu + q\, a_\mu\right) \dot{X}^\mu$$

Can also consider full expansion instead of limit,

$$S = mc^2 \int d\lambda\, \tau_\mu \dot{X}^\mu + \frac{m}{2}\int d\lambda \frac{\bar{h}_{\mu\nu} X^\mu(\lambda) X^\nu(\lambda)}{(\tau_\rho X^\rho(\lambda))^2} + \mathcal{O}(1/c^2)$$

Gives rise to 'type II' torsional Newton-Cartan geometry [see Niels' talk]



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# Non-relativistic strings in curved backgrounds

Now let's do the same kind of limit for Lorentzian Nambu-Goto action,

$$S = -\frac{1}{2\pi\alpha'}\int d^2\sigma \left(\sqrt{-\det \hat{G}_{\alpha\beta}} + \frac{1}{2}\epsilon^{\alpha\beta}\hat{B}_{\alpha\beta}\right)$$

Parametrize $\hat{G}_{MN} = \omega^2 \eta_{AB}\tau_M{}^A\tau_N{}^B + H_{MN}$ , distinguish longitudinal $X^A = (X^0, X^1)$

On worldsheet, longitudinal $\tau_{MN} = \eta_{AB}\tau_M{}^A\tau_N{}^B$ pulls back to Lorentzian metric $\tau_{\alpha\beta}$

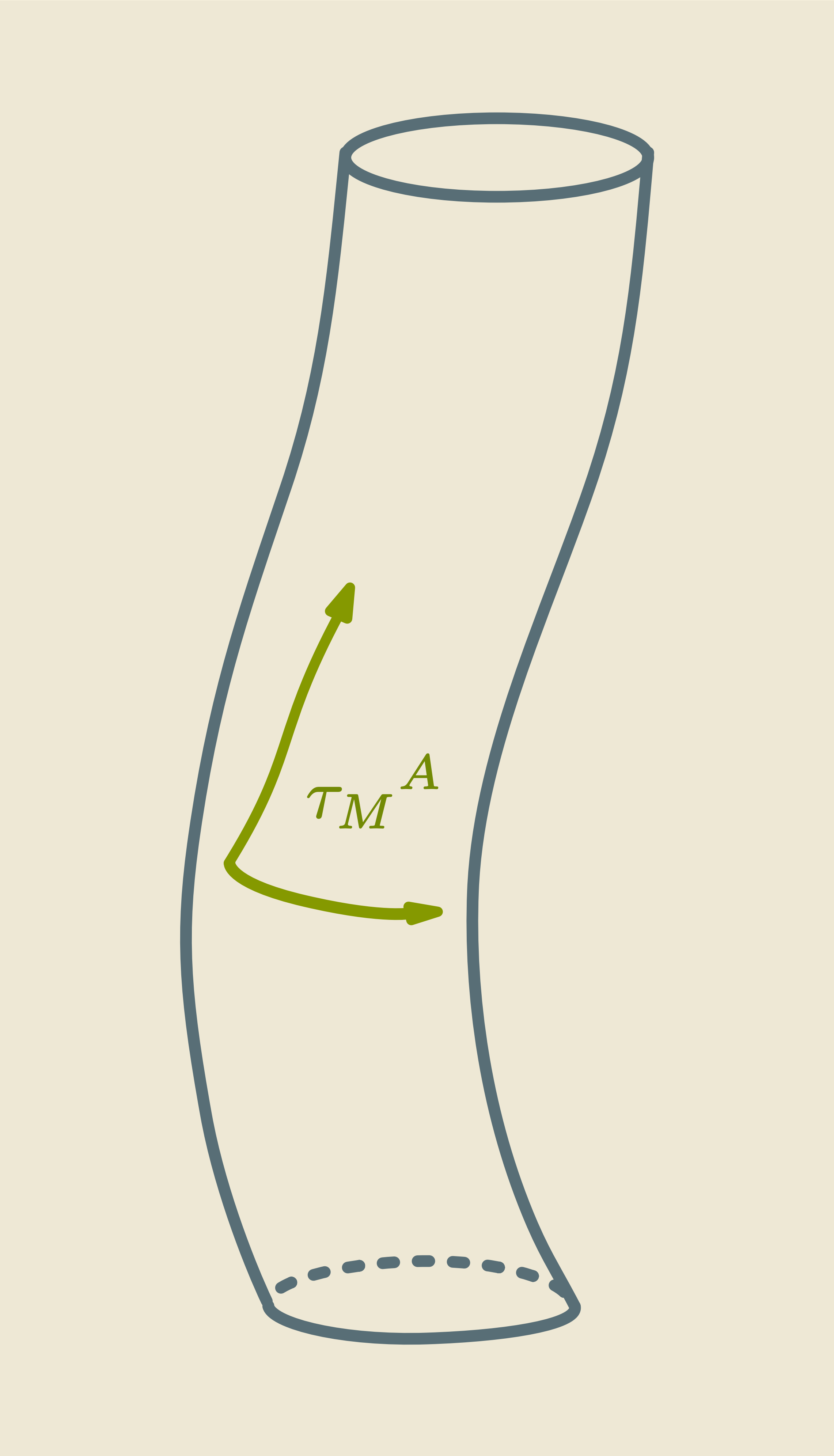
Use this to rewrite metric determinant, [Gomis, Gomis, Kamimura, Townsend]

$$\det \hat{G}_{\alpha\beta} = \omega^4 \det \tau_{\alpha\gamma} \det \left(\delta^\gamma{}_\beta + \frac{1}{\omega^2}\tau^{\gamma\delta}H_{\delta\beta}\right)$$

which results in the Nambu-Goto expansion for $\omega \to \infty$

$$S = -\frac{\omega^2}{2\pi\alpha'}\int d^2\sigma\sqrt{-\det \tau_{\alpha\beta}} - \frac{1}{4\pi\alpha'}\int d^2\sigma\sqrt{-\det \tau_{\alpha\beta}}\tau^{\alpha\beta}H_{\alpha\beta} + \cdots$$

Cancel leading-order term using $\hat{B}_{MN} = -\omega^2\epsilon_{AB}\tau_M{}^A\tau_N{}^B + B_{MN}$ as in point particle



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Parametrize $\hat{G}_{MN} = \omega^2 \eta_{AB}\tau_M{}^A\tau_N{}^B + H_{MN}$ and $\hat{B}_{MN} = -\omega^2\epsilon_{AB}\tau_M{}^A\tau_N{}^B + B_{MN}$,

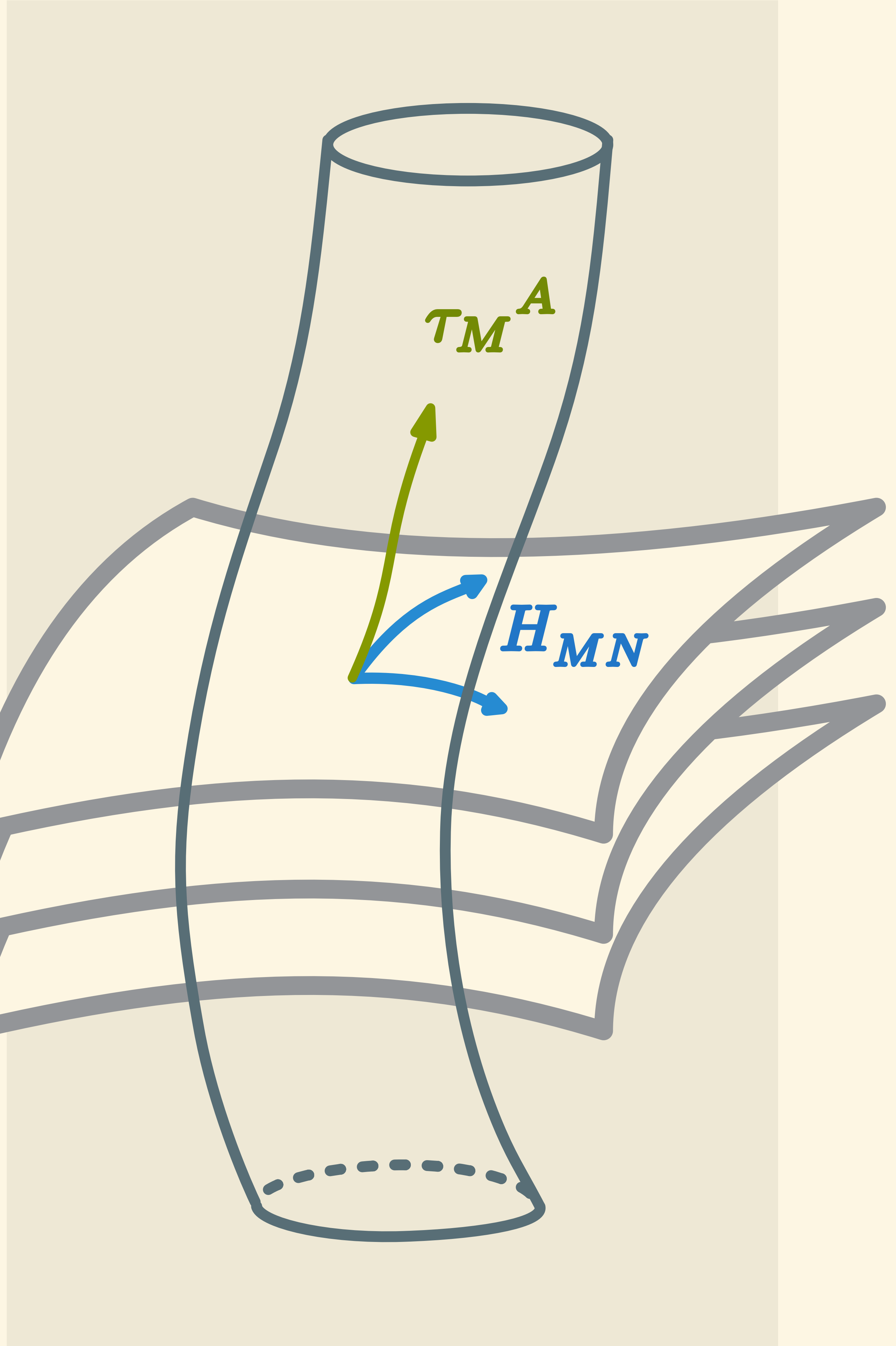
$$S = -\frac{1}{4\pi\alpha'}\int d^2\sigma \left(\sqrt{-\det \tau_{\alpha\beta}}\tau^{\alpha\beta}H_{\alpha\beta} + \epsilon^{\alpha\beta}B_{\alpha\beta}\right)$$

String analogue of particle coupled to type I TNC geometry,

'string Newton-Cartan' geometry (SNC) with 'string Galilei' boosts $\delta X^i \to \Lambda^i{}_A X^A$

[Brugues, Curtright, Gomis, Mezincescu] [Andringa, Bergshoeff, Gomis, De Roo]

- Note that $H_{MN} = H^{\perp}_{MN} + \eta_{AB}\tau_M{}^A m_N{}^B + \eta_{AB}m_M{}^A\tau_N{}^B$
- contains two Bargmann-type fields $m_M{}^A$
- transverse metric $H^{\perp}_{MN}$ and longitudinal $\tau_M{}^A$
- codimension two foliation of spacetime if $d\tau^A = \alpha^A{}_B \wedge \tau^B$



# Non-relativistic strings in curved backgrounds

Nambu-Goto action for non-relativistic strings coupled to string Newton-Cartan (SNC)

$$S = -\frac{1}{4\pi\alpha'}\int d^2\sigma \left(\sqrt{-\det \tau_{\alpha\beta}}\tau^{\alpha\beta}H_{\alpha\beta} + \epsilon^{\alpha\beta}B_{\alpha\beta}\right)$$

with $H_{MN} = H^{\perp}_{MN} + \eta_{AB}\tau_M{}^A m_N{}^B + \eta_{AB}m_M{}^A\tau_N{}^B$

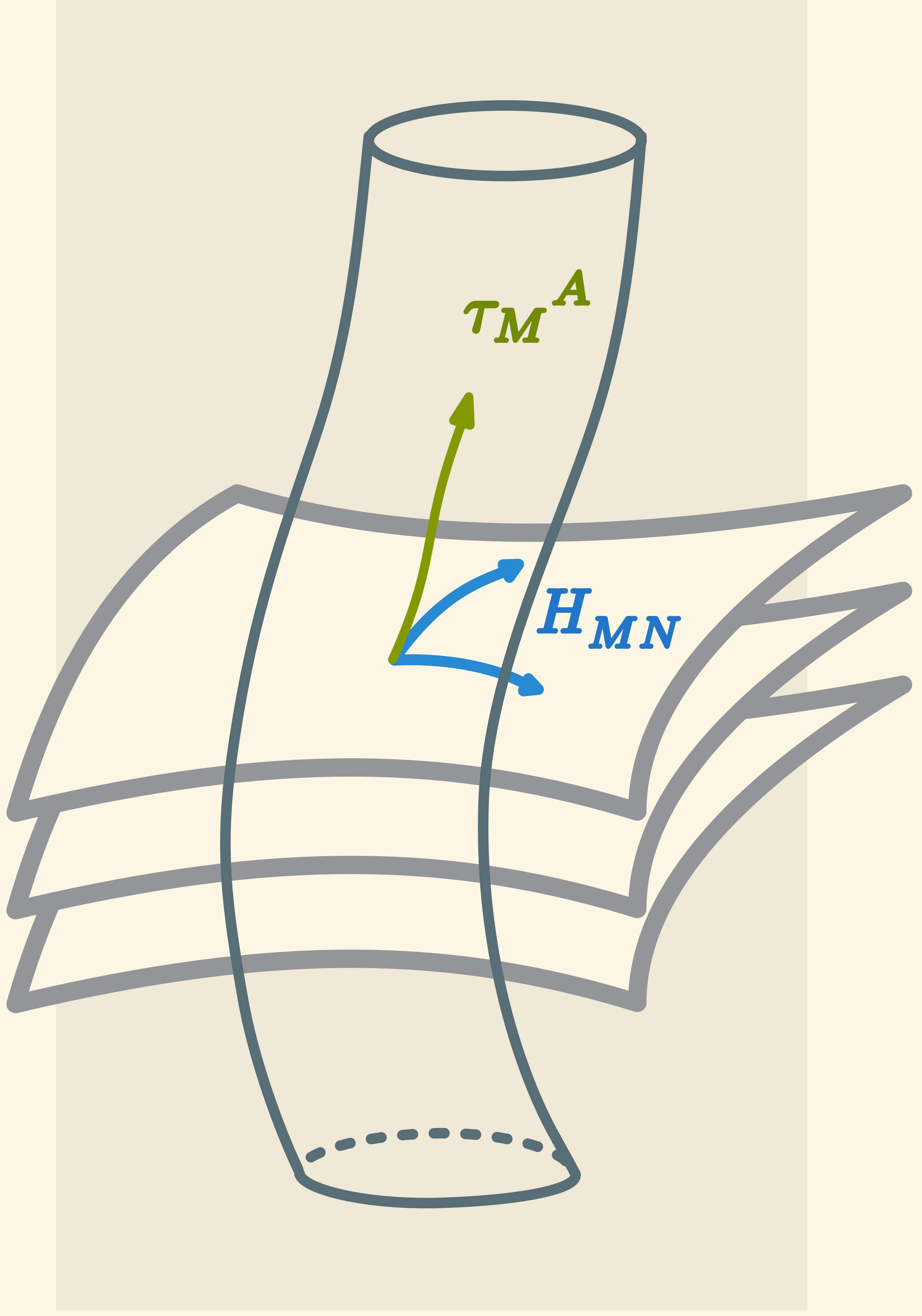
Action contains Stückelberg-type redundancy, [Bergshoeff, Gomis, Rosseel, Şimşek, Yan]

$$H_{\alpha\beta} \to H_{\alpha\beta} + 2\eta_{AB}C_{(\alpha}{}^A\tau_{\beta)}{}^B\,, \qquad B_{\alpha\beta} \to B_{\alpha\beta} + 2\epsilon_{AB}C_{[\alpha}{}^A\tau_{\beta]}{}^B$$

Can keep this redundancy and check final results are invariant under it

Can also remove $\tau_M{}^A$ from $H_{MN} \to H^{\perp}_{MN}$, but then $B_{MN} \to M_{MN}$ *must* contain $\tau_M{}^A$

$$S = -\frac{1}{4\pi\alpha'}\int d^2\sigma \left(\sqrt{-\det \tau_{\alpha\beta}}\tau^{\alpha\beta}H^{\perp}_{\alpha\beta} + \epsilon^{\alpha\beta}M_{\alpha\beta}\right)$$



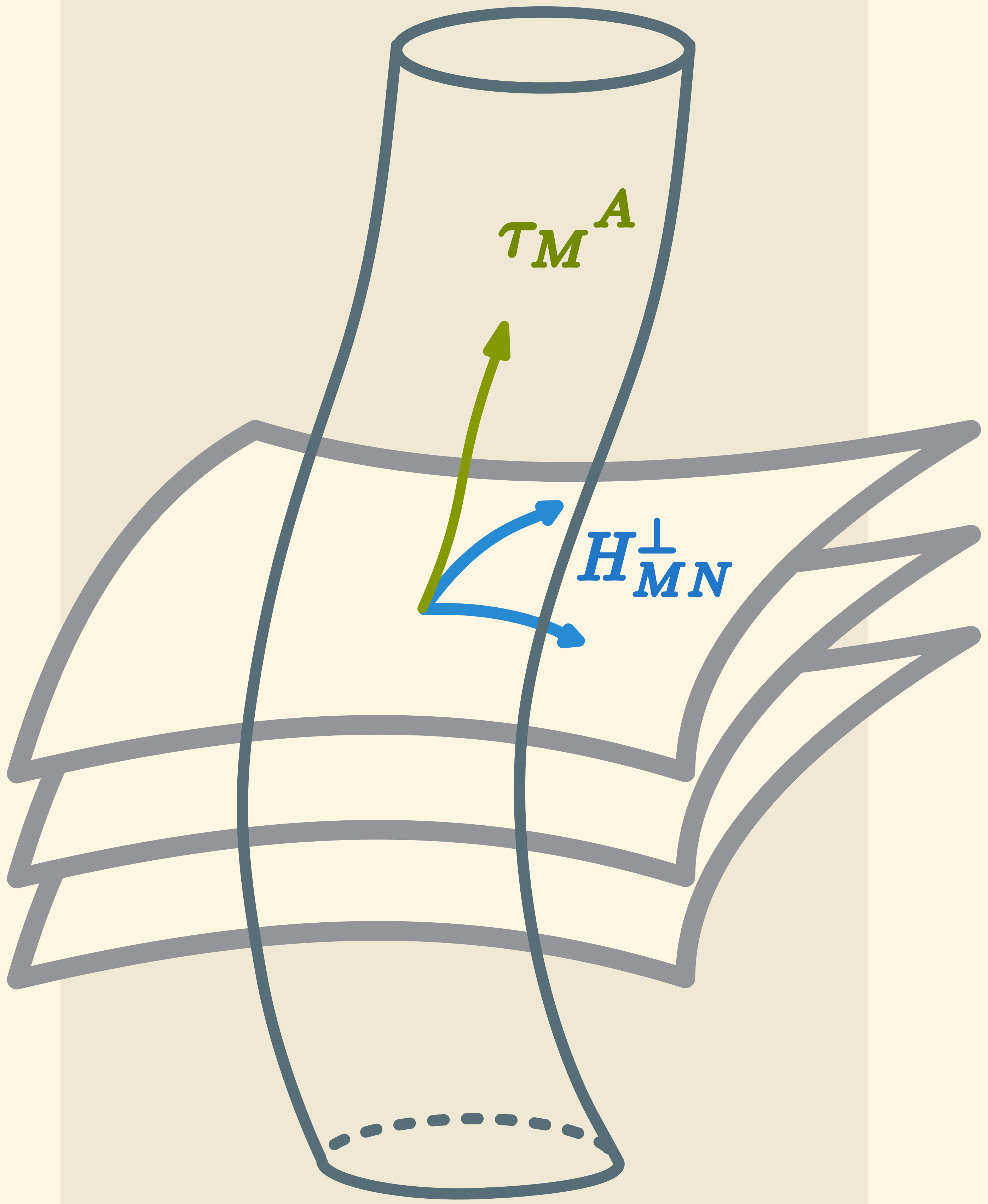
# Non-relativistic strings in curved backgrounds

Nambu-Goto action for non-relativistic strings coupled to string Newton-Cartan (SNC)

$$S = -\frac{1}{4\pi\alpha'}\int d^2\sigma \left(\sqrt{-\det \tau_{\alpha\beta}}\,\tau^{\alpha\beta}H^{\perp}_{\alpha\beta} + \epsilon^{\alpha\beta}M_{\alpha\beta}\right)$$

- 'Kalb-Ramond-type' field $M_{MN}$ transforms under boosts, like $m_\mu$ in TNC
- Action and symmetries can be derived directly from contraction
- Can likewise be obtained from null reduction

[Bidussi, Harmark, Hartong, Obers, Oling]




Alternative approach: double field theory [Ko, Melby-Thompson, Meyer, Park] [Morand, Park]

$$\mathcal{H}_{\underline{MN}} = \begin{pmatrix} \hat{G}^{\mu\nu} & -\hat{G}^{\mu\rho}\,\hat{B}_{\rho\nu} \\ \hat{B}_{\mu\rho}\,\hat{G}^{\rho\nu} & \hat{G}_{\mu\nu} - \hat{B}_{\mu\rho}\,\hat{G}^{\rho\sigma}\,\hat{B}_{\sigma\nu} \end{pmatrix}$$

$$\Longrightarrow \begin{pmatrix} E^{\mu\nu} & -E^{\mu\rho}\,M_{\rho\nu} + \tau^{\mu}{}_{A}\,\tau_{\nu}{}^{B}\,\epsilon^{A}{}_{B} \\ M_{\mu\rho}\,E^{\rho\nu} - \tau_{\mu}{}^{A}\,\tau^{\nu}{}_{B}\,\epsilon_{A}{}^{B} & E_{\mu\nu} - M_{\mu\rho}E^{\rho\sigma}M_{\sigma\nu} - 2\,\tau_{(\mu}{}^{A}\,M_{\nu)\rho}\,\tau^{\rho}{}_{B}\,\epsilon_{A}{}^{B} \end{pmatrix}$$

# Non-relativistic strings in curved backgrounds

Nambu-Goto action for non-relativistic strings coupled to string Newton-Cartan (SNC)

$$S = -\frac{1}{4\pi\alpha'}\int d^2\sigma\left(\sqrt{-\det\tau_{\alpha\beta}}\tau^{\alpha\beta}H^{\perp}_{\alpha\beta} + \epsilon^{\alpha\beta}M_{\alpha\beta}\right)$$

General background geometry has intrinsic torsion $\sim d\tau^A$

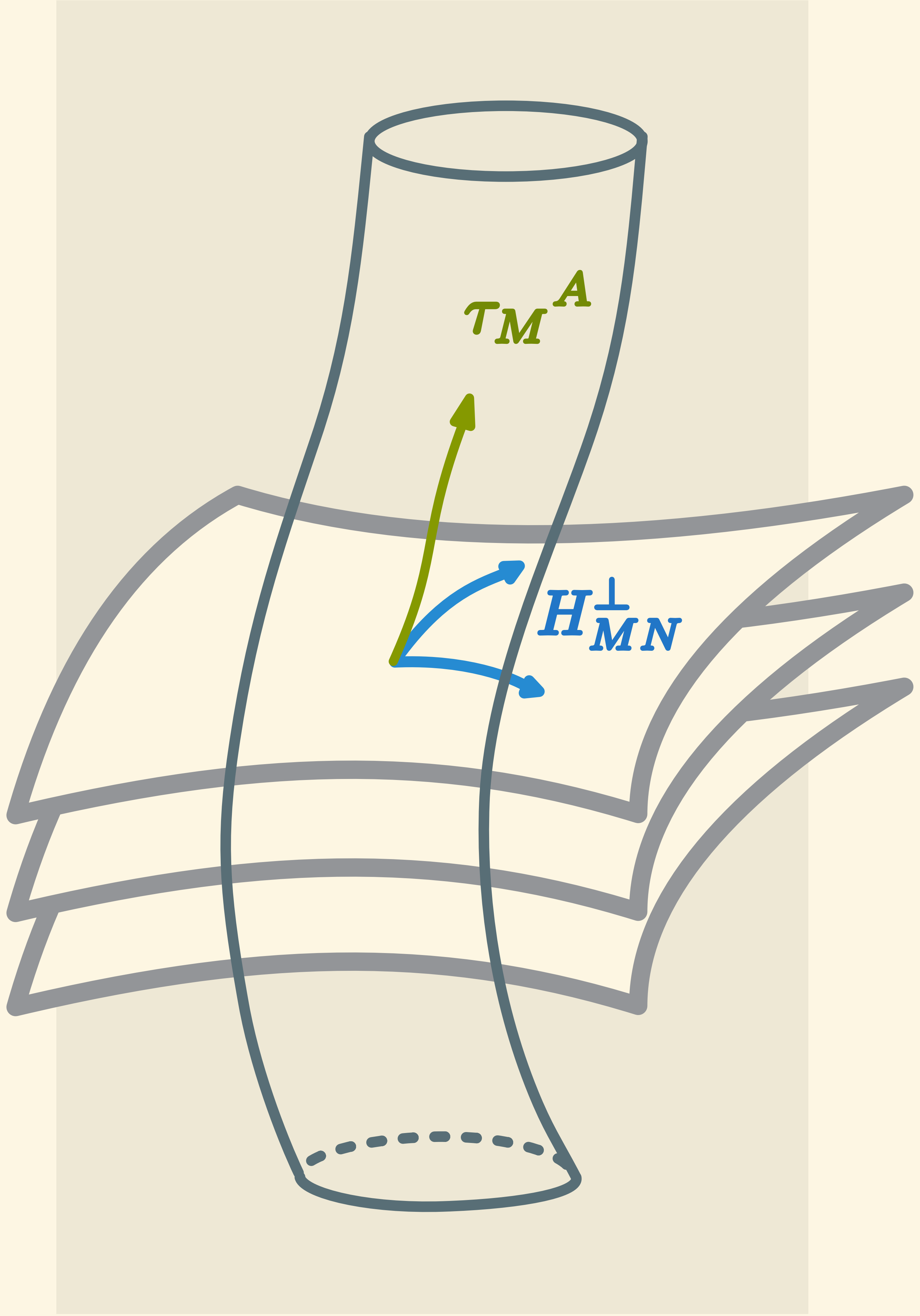
Corresponding Polyakov action with worldsheet vielbeine ${e_\alpha}^A$

$$S = -\frac{1}{4\pi\alpha'}\int d^2\sigma\left(e\,\eta^{AB}{e^\alpha}_A{e^\beta}_B H^{\perp}_{\alpha\beta} + \epsilon^{\alpha\beta}M_{\alpha\beta} + \lambda\,\epsilon^{\alpha\beta}{e_\alpha}^+{\tau_\beta}^+ + \bar\lambda\,\epsilon^{\alpha\beta}{e_\alpha}^-{\tau_\beta}^-\right)$$

Constraints $e^A\wedge\tau^A = 0 \implies {e_\alpha}^A \sim {\tau_\alpha}^A$ up to Lorentz boosts and Weyl transformations

Quantum theory: should not turn on $U(X)\,\lambda\bar\lambda$ coupling, else flow to Lorentzian!

- Beta function $\beta_U = 0$ related to Frobenius condition $d\tau^A = {\alpha^A}_B\wedge\tau^B$
  [Gomis, Oh, Yan, Yu] [Gallegos, Gürsoy, Zinnato]
- Can require action to be invariant under additional $\delta {m_M}^A = D_M\sigma^A$ ,
  only a symmetry if Frobenius condition holds! [Bergshoeff, Gomis, Yan]
- Similar torsion constraints appear in full expansion of string action [Hartong, Have]



# Summary and outlook

Polyakov action for non-relativistic string theory

$$S = -\frac{1}{4\pi\alpha'}\int d^2\sigma \left( e\,\eta^{AB} e^{\alpha}{}_{A} e^{\beta}{}_{B} H^{\perp}_{\alpha\beta} + \epsilon^{\alpha\beta} M_{\alpha\beta} + \lambda\,\epsilon^{\alpha\beta} e_{\alpha}{}^{+} \tau_{\beta}{}^{+} + \bar{\lambda}\,\epsilon^{\alpha\beta} e_{\alpha}{}^{-} \tau_{\beta}{}^{-} + \alpha' R(e)\Phi \right)$$

Beta functions computed, give EOM for effective action
[Gomis, Oh, Yan, Yu] [Gallegos, Gürsoy, Zinnato]

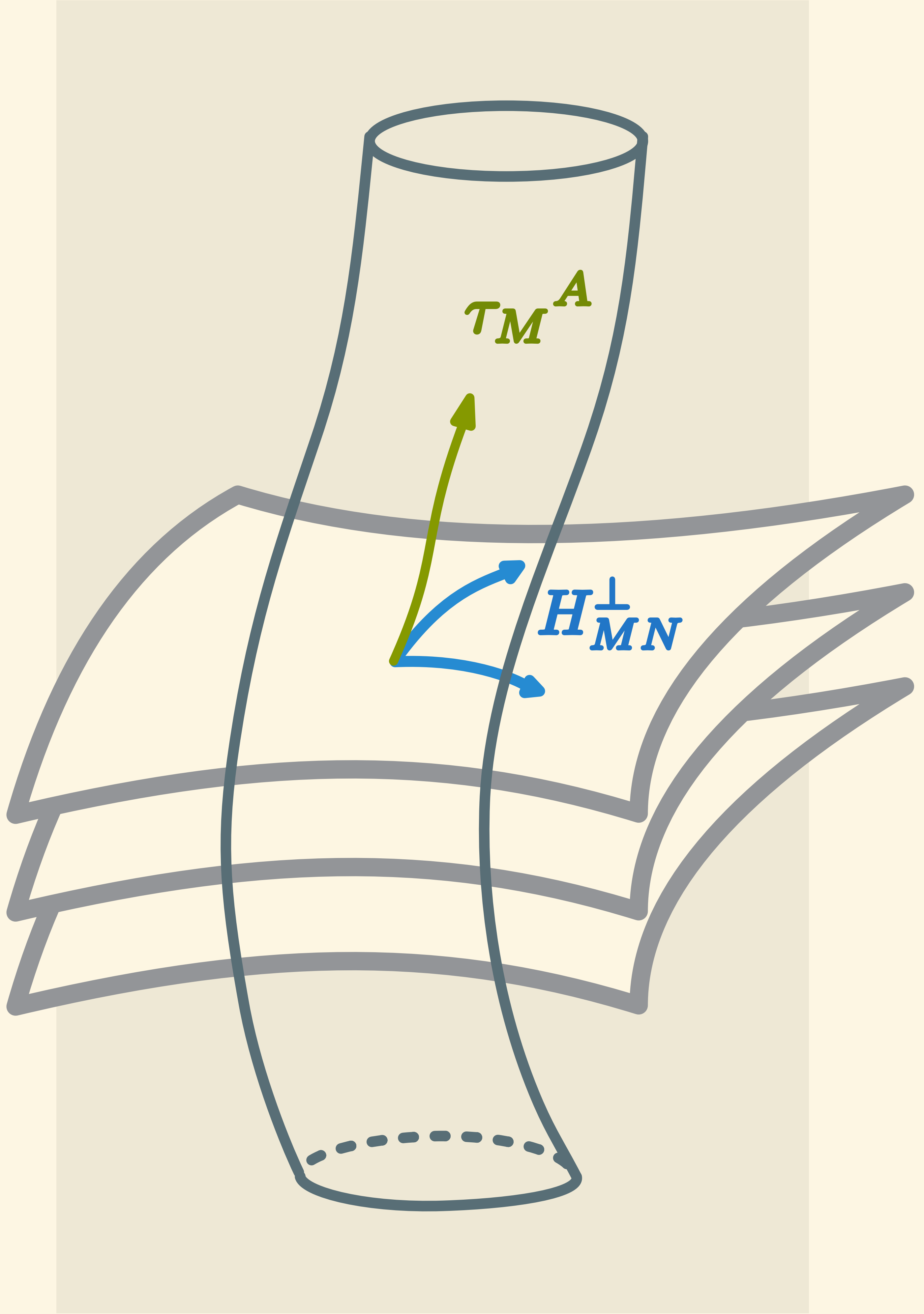
Reproduced from direct limit in target space, also using double field theory
[Bergshoeff, Lahnsteiner, Romano, Rosseel, Şimşek] [Gallegos, Gürsoy, Verma, Zinnato]

Supergravity limit considered, torsion constraints required for finiteness
[Bergshoeff, Lahnsteiner, Romano, Rosseel, Şimşek]

Non-relativistic open string limits and resulting DBI action constructed
[Gomis, Yan, Yu] [Klusoň]

$p$-brane limits, KLT factorization, worldsheet integrability, M5-brane limits…
[Brugues, Curtright, Gomis, Mezincescu] [Pereñiguez] [Roychowdhury] [Gomis, Yan, Yu]
[Fontanella, Nieto-Garcia, Tongeren] [Lambert, Lipstein, Mouland, Orchard, Richmond]

- Expansion perspective [Hartong, Have]
- Closer contact with DLCQ and matrix string theory?



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# Spin Matrix limit in field theory

Spin Matrix Theory: [Harmark, Kristjansson, Orselli]

From $\mathcal{N} = 4$ SYM on $\mathbf{R} \times S^3$ zoom in on BPS bound ($S^3$ isometries $S_i$ and R-charges $J_i$)

$$E \geq Q = \sum a^n S_n + b^n J_n \quad \text{using } \lambda \to 0, \quad N = \text{fixed}, \quad \frac{E - Q}{\lambda} = \text{fixed}$$

Here, focus on $N \to \infty$ and large $Q$ $\Longrightarrow$ sigma models

Example: $SU(2)$ Landau-Lifshitz model from $Q = J_1 + J_2$

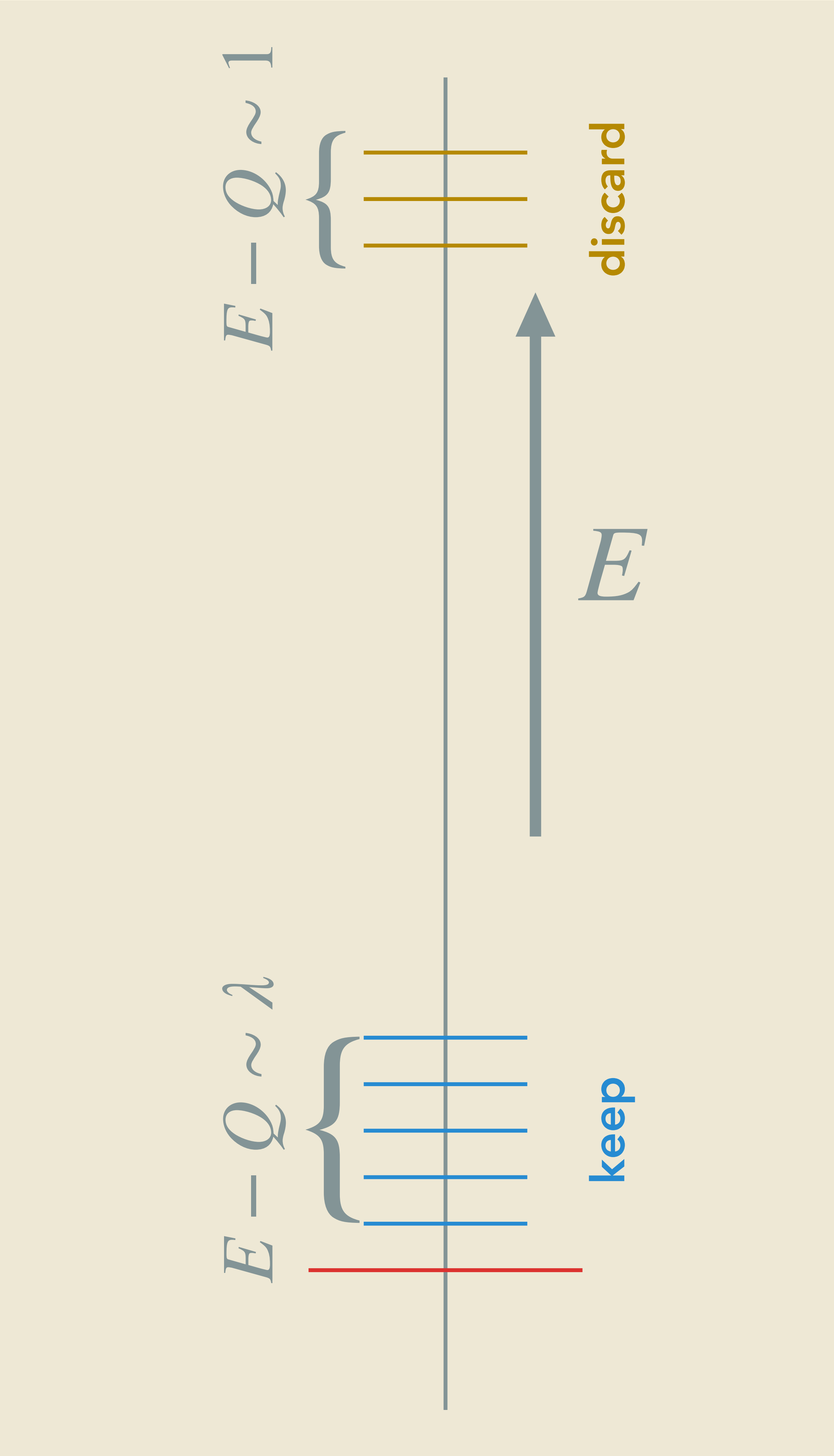
[Kruczenski] [Harmark, Kristjansson, Orselli]

$$S = \frac{Q}{4\pi} \int d^2\sigma \left[ \dot{\varphi} \cos\theta - \frac{1}{4} \left[ (\theta')^2 + \sin^2\theta \left(\varphi'\right)^2 \right] \right]$$

Goal: understand this from *dynamics of non-relativistic string!*

- Where are these directions in $AdS_5 \times S^5$?
- How does non-relativistic behavior arise?
- How to quantize?

[Harmark, Hartong, Obers, Oling Menculini, Yan]



# Spin Matrix limit in string theory

Bulk dual of Spin Matrix limit with $E \geq Q = \sum a^n S_n + b^n J_n$

$$g_s \to 0, \qquad N = \text{fixed}, \qquad \frac{E-Q}{g_s} = \text{fixed}$$

Procedure: [Harmark-Hartong-Obers]

- find a combination of angles $\gamma$ such that $Q = -i\partial_\gamma$
- define $x^0 = (t+\gamma)/2$ and $u = \gamma - t$ and rescale $x^0 \sim \tilde{x}^0/g_s$

$$i\partial_{\tilde{x}^0} = \frac{E-Q}{g_s} \text{ and } -i\partial_u = (E+Q)/2$$

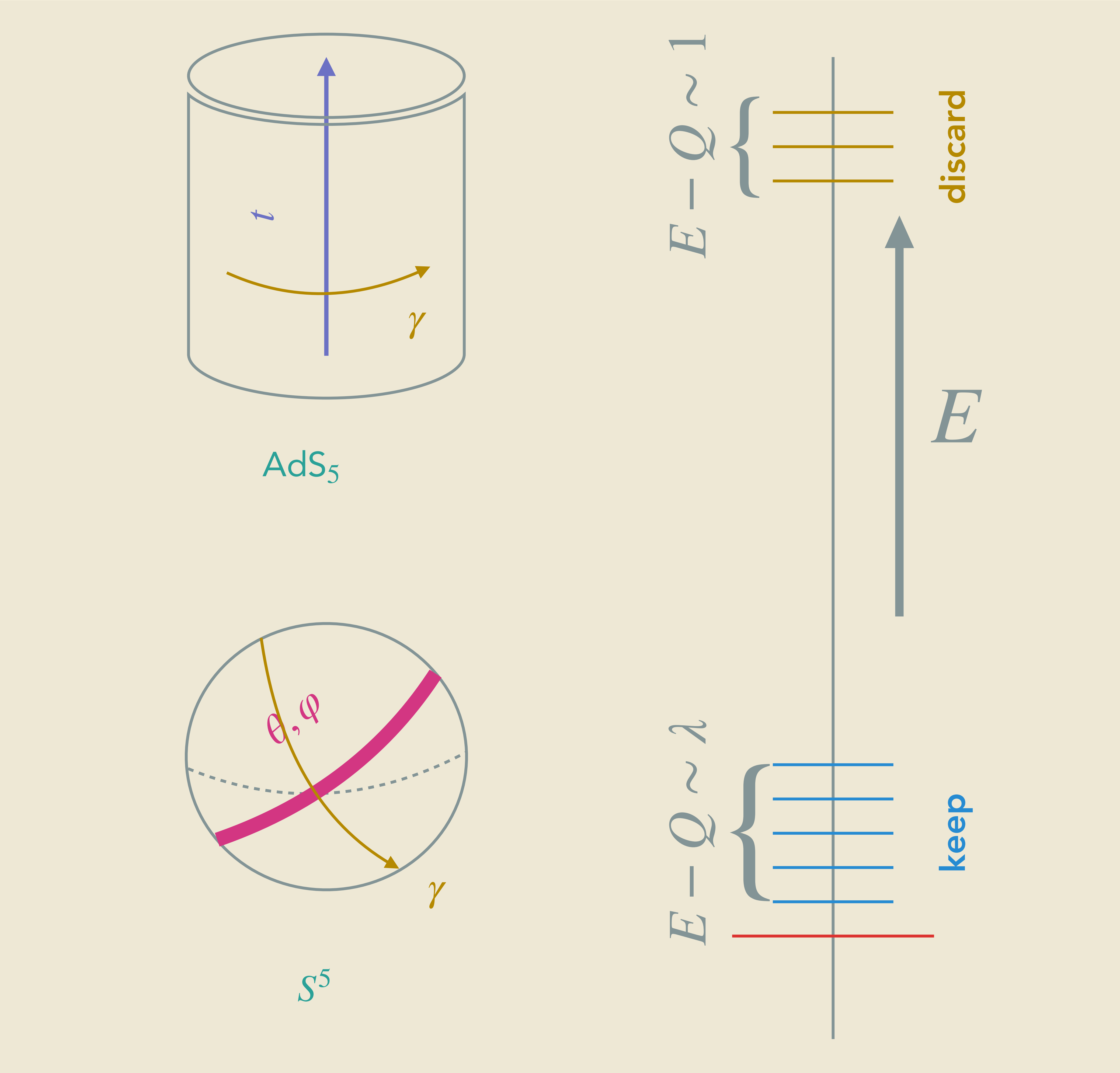
- keeps only dynamics on submanifold with $\partial_u$ is null

Example: $SU(2)$ Spin Matrix string from $Q = J_1 + J_2$

To get $Q = -i\partial_\gamma$ parametrize $S^5$ using Hopf coordinates

Then restrict to $\rho = 0$ in $AdS_5$ and $\beta = \pi$ in $S^5$

$$ds^2\Big|_M \implies \tilde{\tau} = d\tilde{x}^0, \quad m = -\frac{R^2}{2}\cos\theta d\varphi, \quad h = \frac{R^2}{4}\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$



# Spin Matrix limit in string theory

Nambu-Goto action for non-relativistic strings on SNC background

$$S = -\frac{1}{4\pi\alpha'}\int d^2\sigma\left(\sqrt{-\tau}\tau^{\alpha\beta}H^{\perp}_{\alpha\beta} + \epsilon^{\alpha\beta}M_{\alpha\beta}\right)$$

Rescale ${\tau_M}^0 = c\,{\tilde{\tau}_M}^0$ , $\tau^1 = \tilde{\tau}^1$ and $\alpha' = c\,\tilde{\alpha}'$ , $M_{MN} = c\,\tilde{M}_{MN}$ where $c = \frac{1}{\sqrt{4\pi g_s N}} \to \infty$

$$S = -\frac{1}{4\pi\tilde{\alpha}'}\int d^2\sigma\left(\sqrt{-\tau}\tilde{\tau}^{\alpha}{}_1\tilde{\tau}^{\alpha}{}_1 H^{\perp}_{\alpha\beta} + \epsilon^{\alpha\beta}\tilde{M}_{\alpha\beta}\right)$$

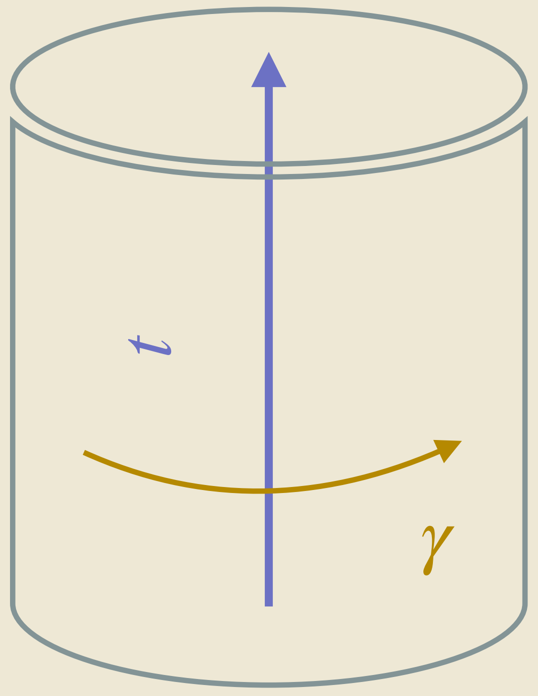
Gives Galilean structure $\left({\tilde{\tau}_\alpha}^0, {\tilde{\tau}_\alpha}^1\right)$ on worldsheet!

Polyakov action for non-relativistic string theory

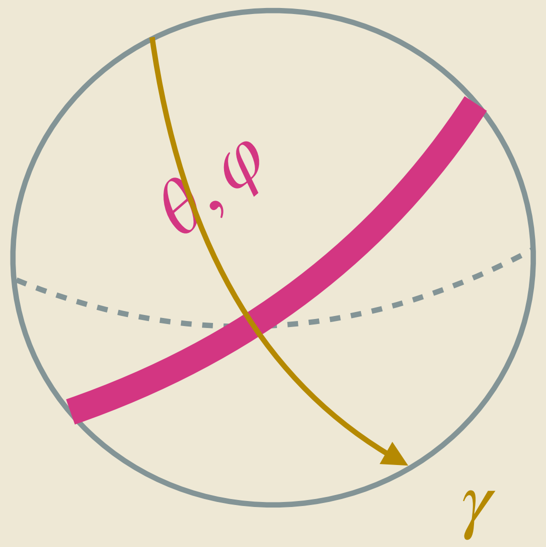
$$S = -\frac{1}{4\pi\alpha'}\int d^2\sigma\left(e\,\eta^{AB}e^{\alpha}{}_A e^{\beta}{}_B H^{\perp}_{\alpha\beta} + \epsilon^{\alpha\beta}M_{\alpha\beta} + \lambda\,\epsilon^{\alpha\beta}{e_\alpha}^+{\tau_\beta}^+ + \bar{\lambda}\,\epsilon^{\alpha\beta}{e_\alpha}^-{\tau_\beta}^-\right)$$

Now rescale ${e_\alpha}^0 = c\,{\tilde{e}_\alpha}^0$ , ${e_\alpha}^1 = {\tilde{e}_\alpha}^1$ and $\lambda^0 = \tilde{\lambda}^0/2c$ , $\lambda_0 = \tilde{\lambda}^1/2$

$$S = -\frac{1}{4\pi\tilde{\alpha}'}\int d^2\sigma\left(\tilde{e}\,\tilde{e}^{\alpha}{}_1\tilde{e}^{\beta}{}_1 H^{\perp}_{\alpha\beta} + \epsilon^{\alpha\beta}\tilde{M}_{\alpha\beta} + \tilde{\lambda}^0\epsilon^{\alpha\beta}{\tilde{e}_\alpha}^0{\tau_\beta}^0 + \tilde{\lambda}^1\epsilon^{\alpha\beta}\left[{\tilde{e}_\alpha}^0{\tilde{\tau}_\beta}^1 + {\tilde{e}_\alpha}^1{\tilde{\tau}_\beta}^0\right]\right)$$




$AdS_5$




$S^5$

# Spin Matrix limit in string theory

Spin Matrix strings Polyakov action

$$S = -\frac{1}{4\pi\tilde{\alpha}'}\int d^2\sigma \left( \tilde{e}\, \tilde{e}^{\alpha}{}_1 \tilde{e}^{\beta}{}_1 H^{\perp}_{\alpha\beta} + \epsilon^{\alpha\beta}\tilde{M}_{\alpha\beta} + \tilde{\lambda}^0\, \epsilon^{\alpha\beta}\tilde{e}_{\alpha}{}^0 \tau_{\beta}{}^0 + \tilde{\lambda}^1\, \epsilon^{\alpha\beta}\left[\tilde{e}_{\alpha}{}^0\tilde{\tau}_{\beta}{}^1 + \tilde{e}_{\alpha}{}^1\tilde{\tau}_{\beta}{}^0\right]\right)$$

Constraints $e^0 \wedge \tau^0 = 0$ and $e^0 \wedge \tau^1 + e^1 \wedge \tau^0 = 0$ fix Galilean structure on worldsheet

up to Weyl transformations $e^A \to \Omega\, e^A$ and Galilean boosts $e^0 \to e^0$ , $e^1 \to e^1 + \gamma\, e^0$

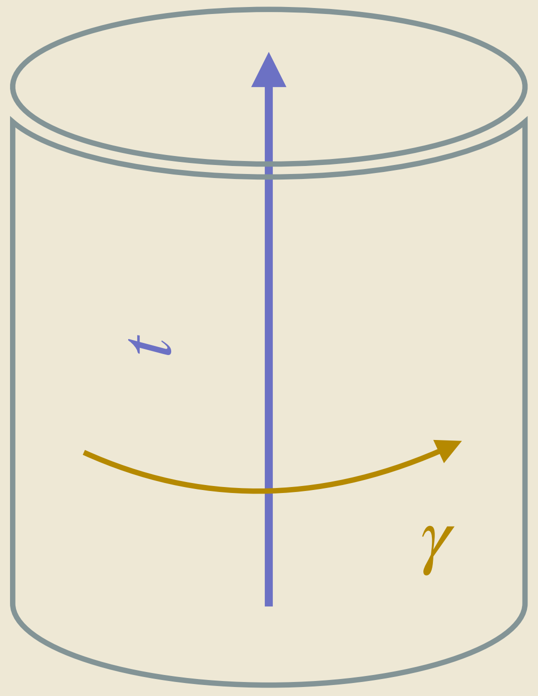
In flat gauge $e^0 = d\sigma^0$ , $e^1 = J\, d\sigma^1$ get residual Galilean conformal algebra (GCA),

*not* Virasoro symmetry, no longer $CFT_2$ on worldsheet!

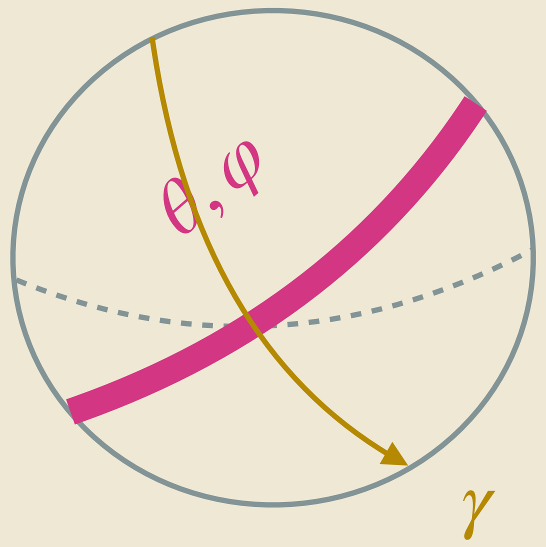
Fixing GCA with $X^0 = J^2\, \sigma^0$ , $X^1 = J\, \sigma^1$ reproduces $SU(2)$ Landau-Lifshitz action

$$S = -\frac{J}{2\pi}\int d^2\sigma \left[m_i \dot{X}^i + H^{\perp}_{ij}\dot{X}^i\dot{X}^j\right] = \frac{J}{4\pi}\int d^2\sigma \left[\dot{\varphi}\cos\theta - \frac{1}{4}\left[(\theta')^2 + \sin^2\theta\,(\varphi')^2\right]\right]$$

on background determined by limit




AdS$_5$




$S^5$

# Spin Matrix limit in string theory

Easier sigma model? Take $SU(2)$ Spin Matrix string from $Q = J_1 + J_2$

$$\tau = d\tilde{x}^0, \quad m = -\frac{1}{2}\cos\theta d\varphi, \quad h = \frac{1}{4}\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

Simplify by taking $Q \to \infty$ with $\tilde{x}^0$ fixed and $$u = \frac{\tilde{u}}{Q} \quad \theta = \frac{\pi}{2} + \frac{x}{\sqrt{Q}}, \quad \varphi = \frac{y}{\sqrt{Q}}$$

This leads to the 'flat' background $$\tau = d\tilde{x}^0, \quad m = \frac{1}{2}xdy, \quad h = \frac{1}{4}\left(dx^2 + dy^2\right)$$

and the 'light-cone' string action $$S = \frac{1}{4\pi}\int d^2\sigma \left(x\dot{y} - \frac{1}{4}\left[(x')^2 + (y')^2\right]\right)$$

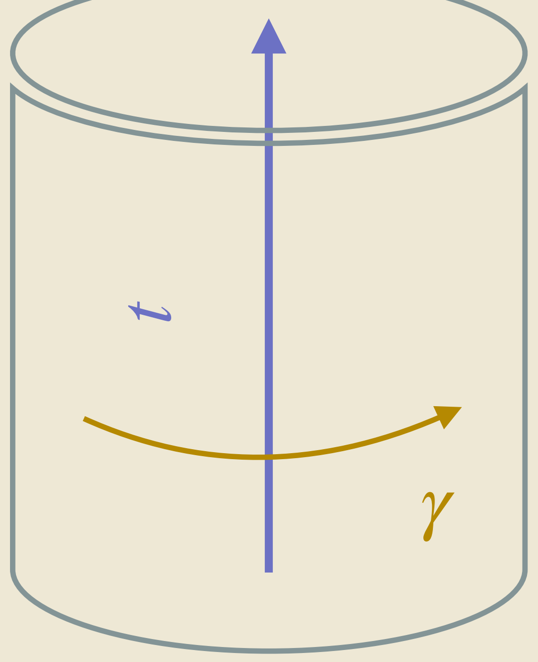
Penrose limit $Q \to \infty$ of $AdS_5 \times S^5$ gives pp-wave geometry

$$ds^2 = 2dx^0du - 2m_\alpha dx^\alpha du + d\mathbf{x}^2 - \delta_{ij}x^ix^j\left(dx^0\right)^2$$

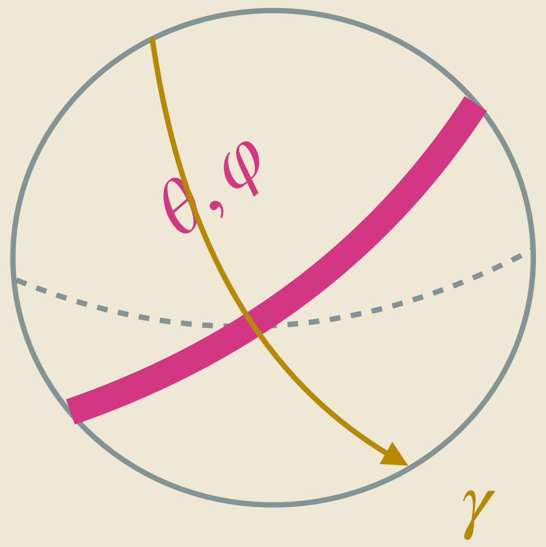
Split coordinates $(u, x^0, x^\alpha, x^i)$ where [Bertolini ea., Grignani ea.]

- $x^i$ feel quadratic potential $\Longrightarrow$ decouple in SMT limit
- $x^\alpha$ are 'flat' $\Longrightarrow$ parametrize SMT dynamics

*Agrees with 'flat limit'* $Q \to \infty$ of 'curved' U(1)-Galilean!




$AdS_5$




$S^5$

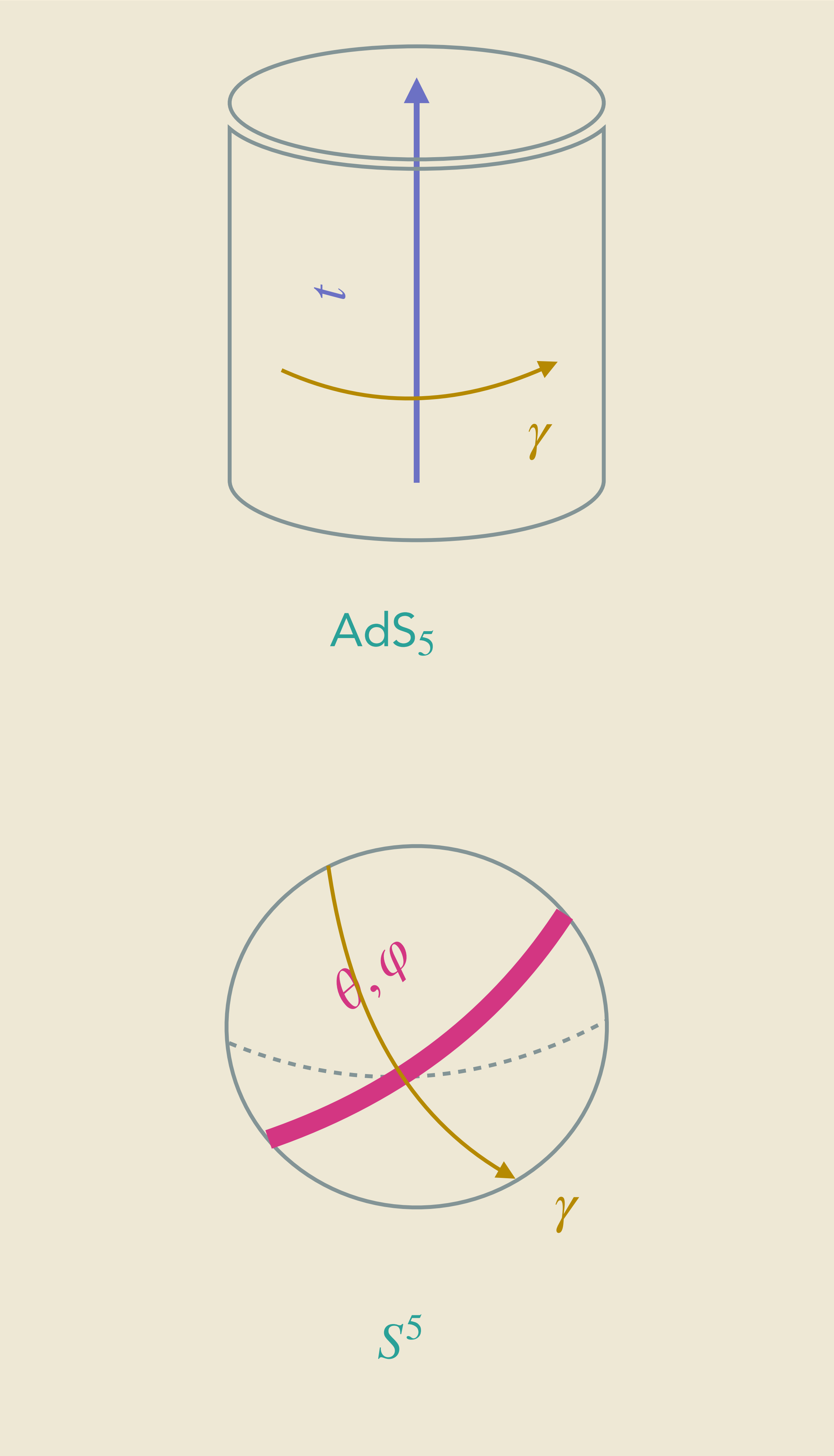
# Spin Matrix limit in string theory

Penrose and SMT limit *commute!*

rel. string on $AdS_5 \times S^5$ — $g_s \to 0$ (SMT limit) → SMT string on curved NR background

Penrose limit: $Q \to \infty$ ↓  $Q \to \infty$ ↓ `flat' limit

rel. string on pp-wave — $g_s \to 0$ (SMT limit) → SMT string on 'flat' NR background



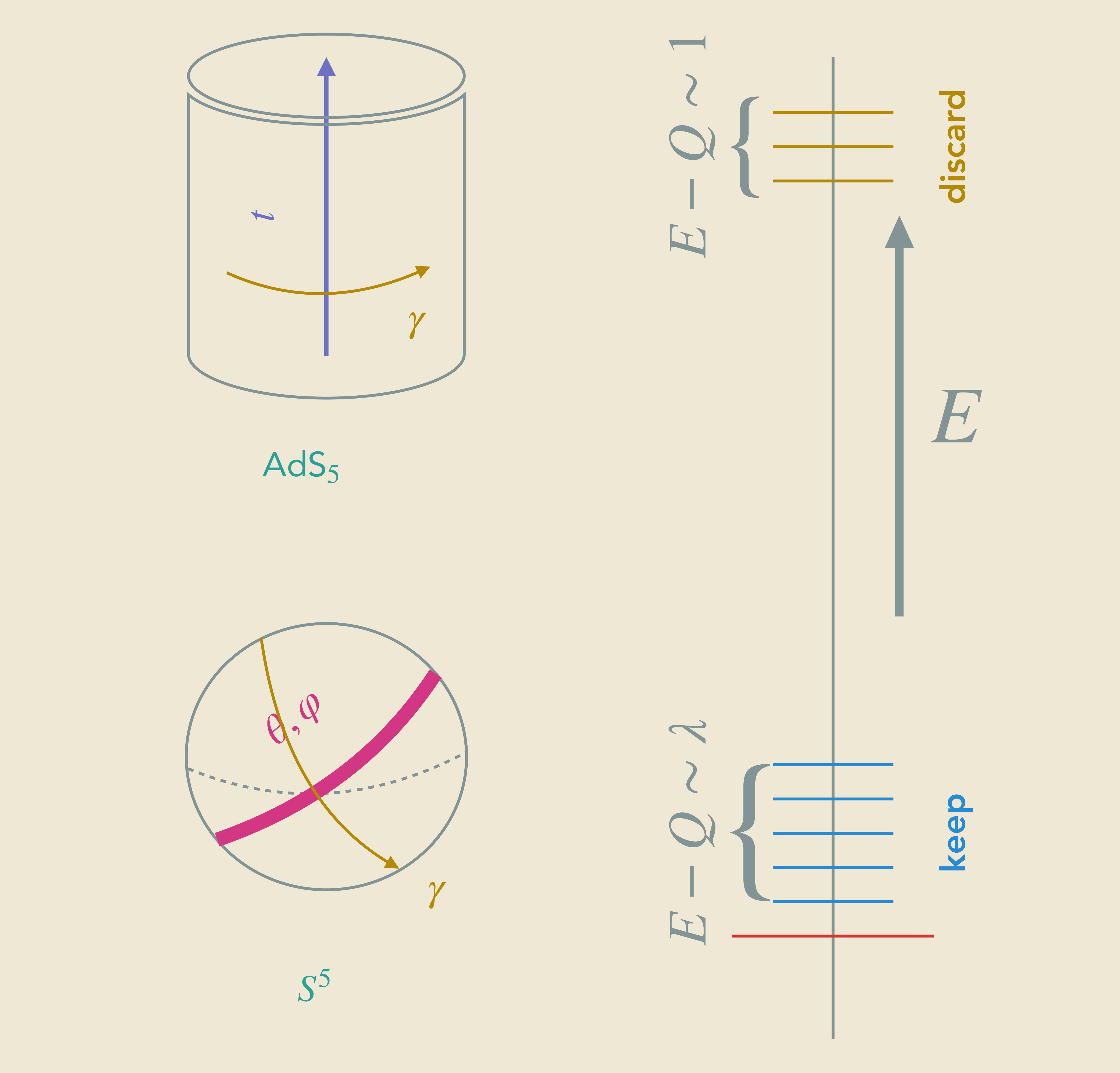
# Summary and outlook

Spin Matrix limit can be mapped to strings in $\mathrm{AdS}_5 \times S^5$

Leads to strings with non-Lorentzian worldsheet geometry

GCA instead of Virasoro, no longer $\mathrm{CFT}_2$ on worldsheet

- Quantization of worldsheet
- NR holography with recent field theory results?
- 1/16 BPS black hole microstates in $PSU(1,2|3)$ limit?
- Beyond $N \to \infty$ in bulk? Dilaton term?
- Similar limit for $AdS_3 \times S^3 \times \mathbb{T}^4$ or $AdS_3 \times S^3 \times S^3 \times S^1$ ?



# Outline

- Introduction: Gomis-Ooguri limit
- Warmup: non-relativistic point particle
- Gomis-Ooguri strings in curved backgrounds
- Spin Matrix limits of strings on AdS
- Outlook

# Outlook

String Newton-Cartan geometry

gives covariant formulation of non-relativistic string theory

describes closed subsector of relativistic string theory

*What can we add to 90's string theory knowledge?*

- Covariant (better?) formulation of DLCQ for strings?
- Further contact with matrix string theory?
- Expansion beyond non-relativistic limit?