Carroll Approximations of GR, BKL Dynamics and Holography

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Introduction

Kasner geometries

$$ds^2 = -\,dt^2 + t^{2p_x}\,dx^2 + t^{2p_y}\,dy^2 + t^{2p_z}\,dz^2$$

homogeneous and anisotropic solution to Einstein equations

Adding spatial curvature or matter:

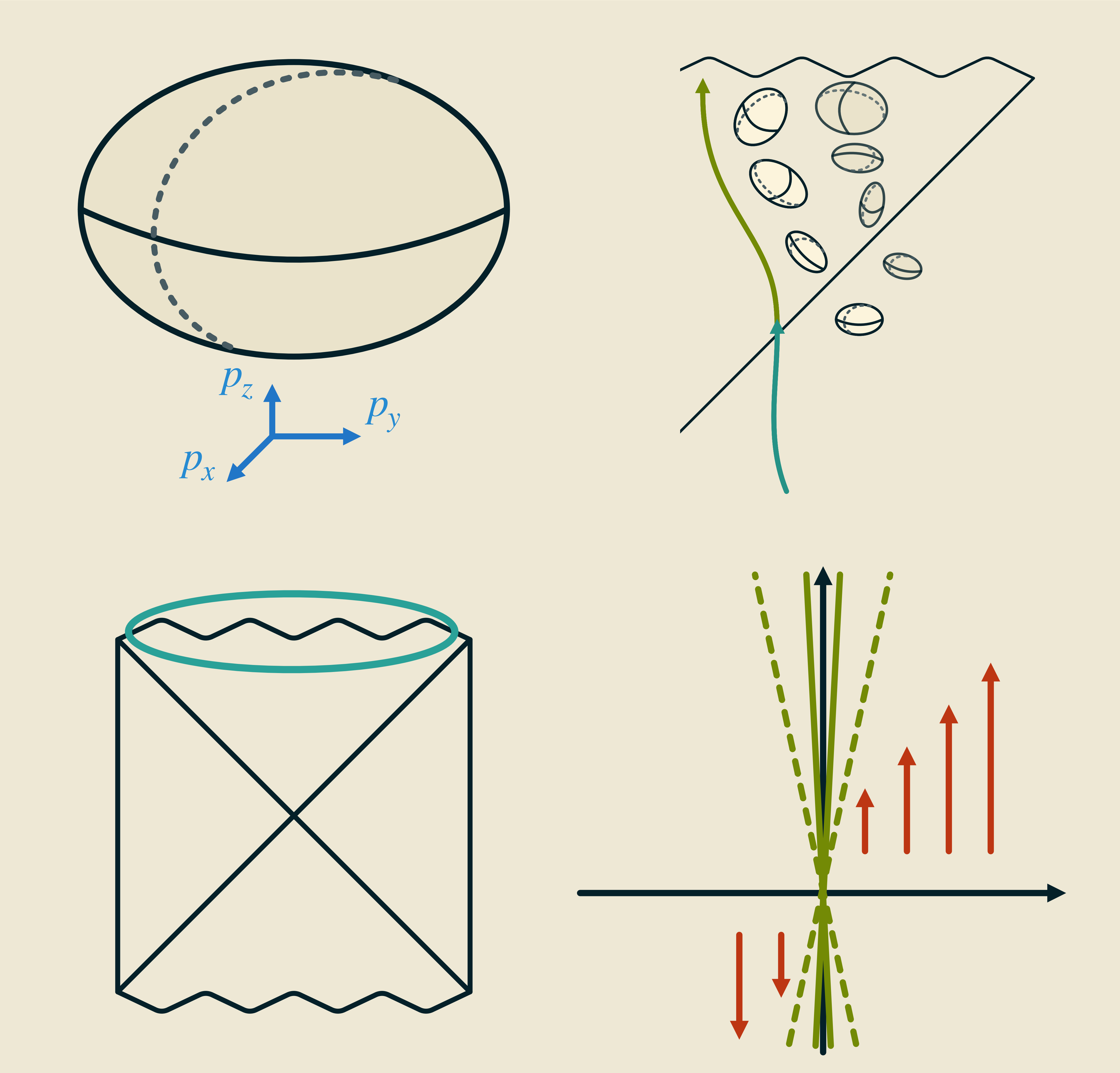
rich and possibly chaotic dynamics in space of allowed p_i

Belinski–Khalatnikov–Lifshitz (BKL) conjecture:

near spacelike singularities, the generic behavior of GR is given by ultra-local chaotic dynamics of this type

Motivations:

- Difficult regime in GR, study using Carroll approximation?
- How does AdS/CFT encode singularity and BKL dynamics?



Outline

- Kasner and BKL in gravity
- Carroll limits and geometry
- Carroll approximation of general relativity
- Mixmaster from Carroll gravity

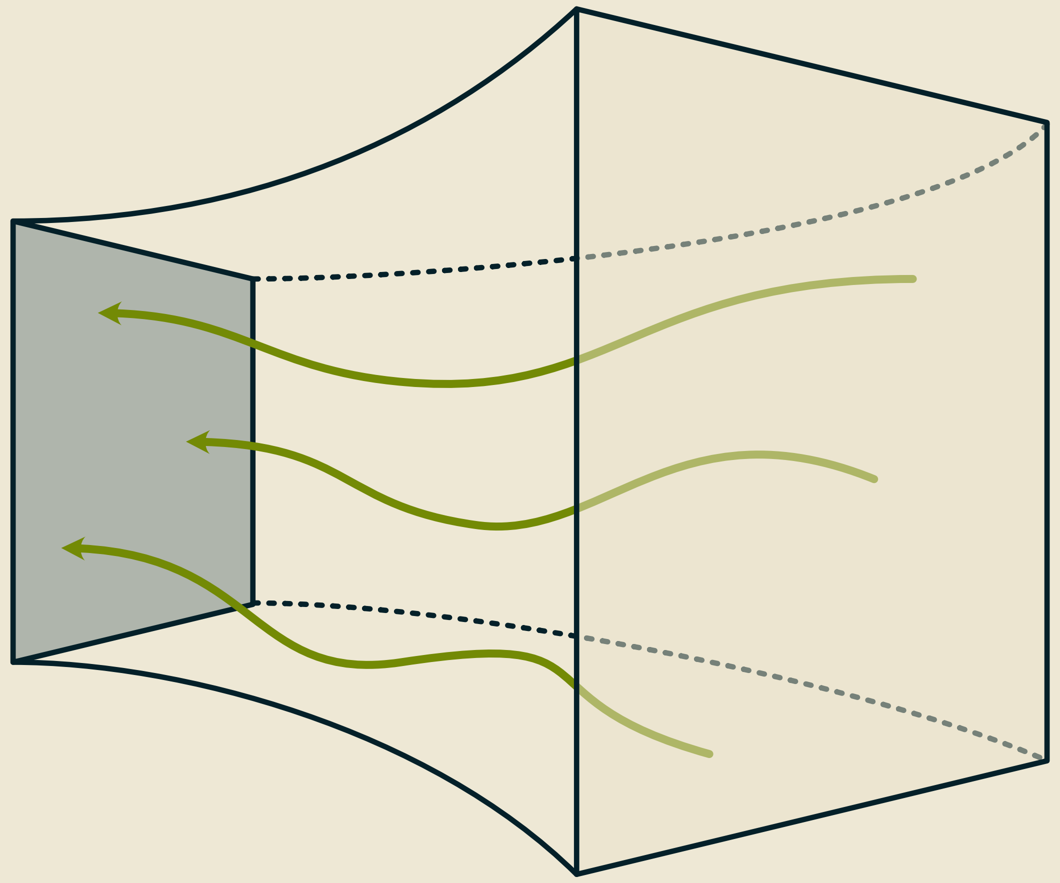
Kasner geometries in GR

Take planar AdS black hole and zoom in behind horizon, $f(z) = 1 - (z/z_H)^3 \approx -(z/z_H)^3$

$$ds^2 = \frac{1}{z^2}\left[-f(z)dt^2 + \frac{dz^2}{f(z)} + dx^2 + dy^2\right]$$

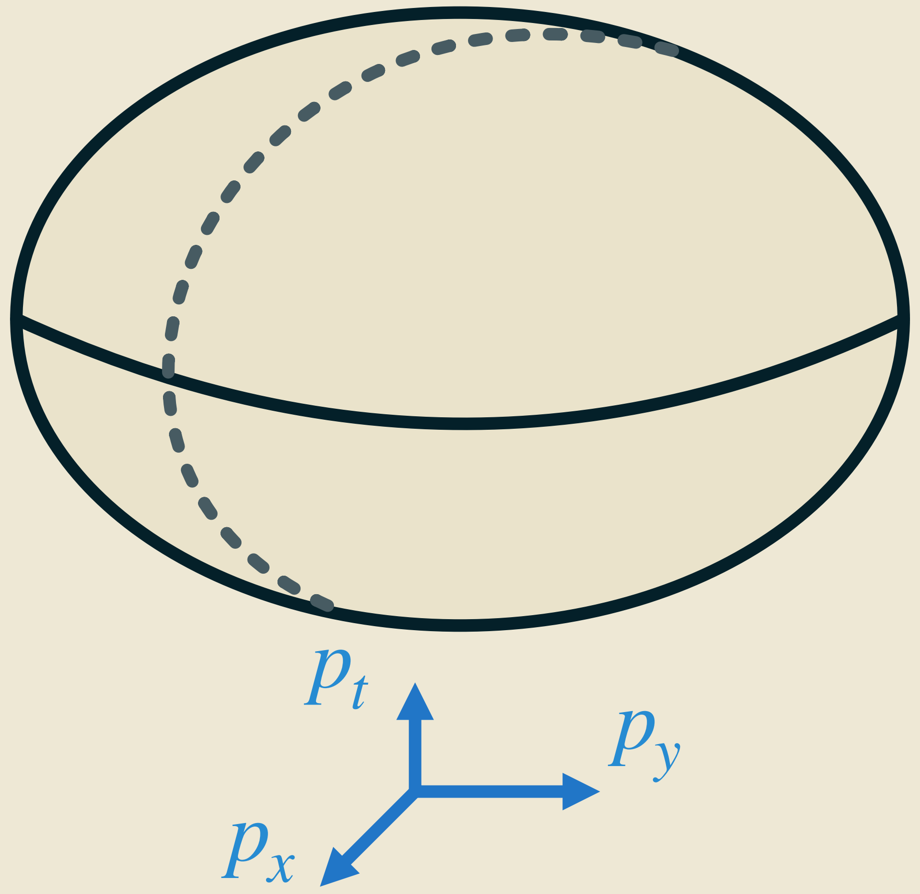
$$\approx -d\tau^2 + \#\frac{dt^2}{\tau^{2/3}} + \#\tau^{4/3}\left(dx^2 + dy^2\right)$$

where $\tau = \tau(z)$ is interior 'radial time'



Example of Kasner geometry with $p_t = -1/3$ and $p_x = p_y = 2/3$

$$ds^2 = -d\tau^2 + \tau^{2p_t}dt^2 + \tau^{2p_x}dx^2 + \tau^{2p_y}dy^2$$



Solution of vacuum Einstein equations if $\sum p_i = 1$ and $\sum (p_i)^2 = 1$

Kasner geometries in GR

Parametrize Kasner solutions using lapse $\alpha(t)$ and scaling exponents $\beta_i(t)$

$$ds^2 = -\,e^{-2\alpha(t)}dt^2 + e^{2\beta_x(t)}dx^2 + e^{2\beta_y(t)}dy^2 + e^{2\beta_z(t)}dz^2$$

Vacuum Einstein equations then give

$$0 = E_{tt} = \dot{\beta}^T \begin{pmatrix} 0 & -1 & -1 \\ -1 & 0 & -1 \\ -1 & -1 & 0 \end{pmatrix} \dot{\beta} = -\left(\dot{\bar{\beta}}_1\right)^2 + \left(\dot{\bar{\beta}}_2\right)^2 + \left(\dot{\bar{\beta}}_3\right)^2$$

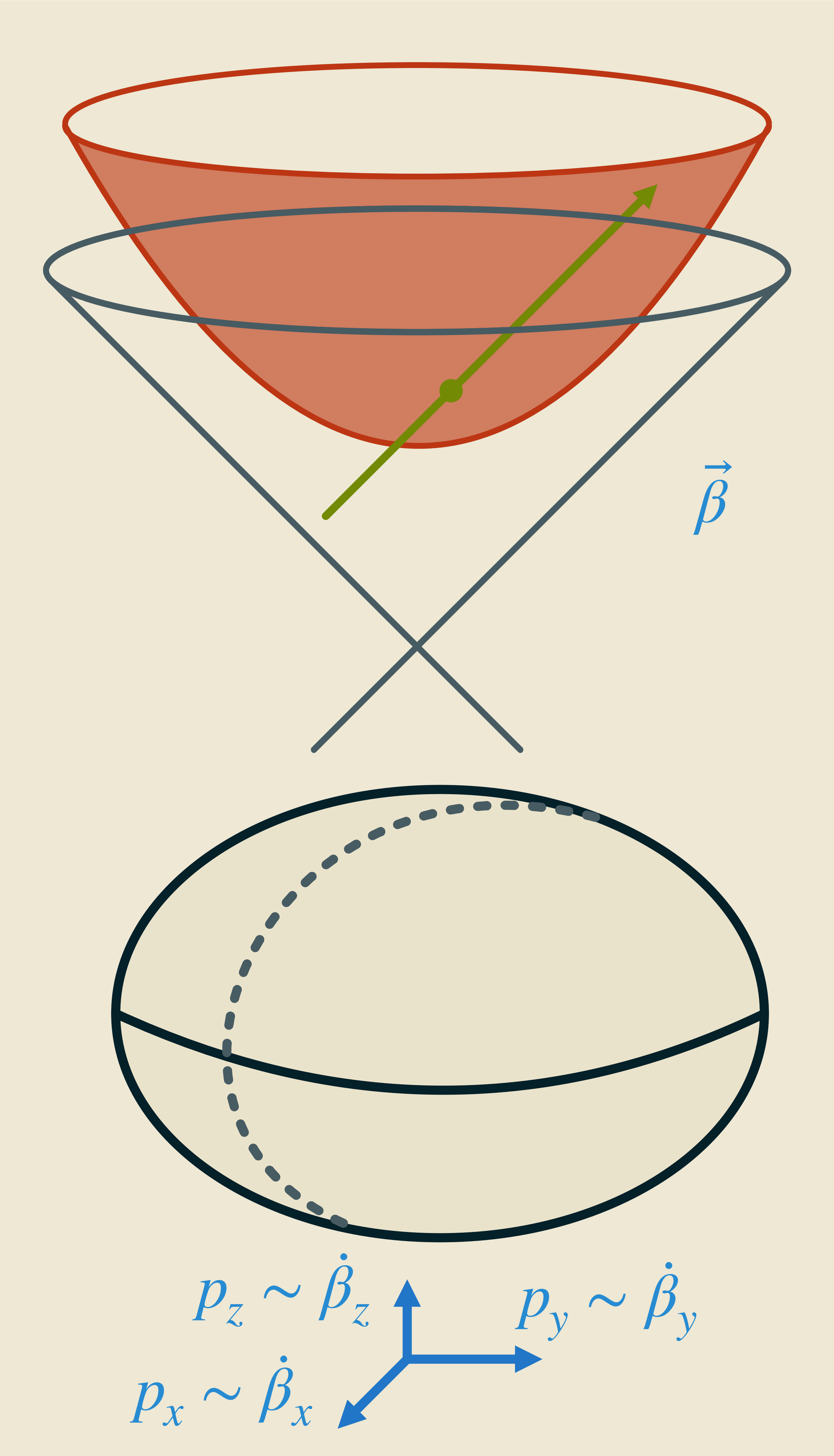
Interpret $\dot{\beta}_i(t)$ as null vector in Minkowski superspace! [Chitre] [Damour, Henneaux, Nicolai]

After shifting lapse and reparametrizing $t = t(\tau)$, spatial components $E_{ii} = 0$ give

$$0 = \ddot{\beta}_i(\tau) \qquad \Longrightarrow \qquad \beta_i = \beta_i^{(0)} + v_i \tau$$

so $\beta_i(\tau)$ parametrizes null geodesic in $\mathbb{R}^{1,2}$ superspace!

Maps to (so far simple) particle motion on future hyperboloids $\mathbb{H}^2 \subset \mathbb{R}^{1,2}$



Mixmaster dynamics in GR

Interesting dynamics from spatial curvature and/or matter coupling

Example: $SO(3)$ group manifold ('mixmaster/Bianchi IX') [Misner]

Homogeneous isotropic metric from Maurer-Cartan forms μ^i

$$d\sigma^2 = \delta_{ij}\mu^i\mu^j = d\varphi^2 + 2\cos\theta\, d\varphi d\psi + d\theta^2$$

Homogeneous anisotropic space using scaling exponent matrix $\beta_{ij}(t) = \beta_i(t)\,\delta_{ij}$

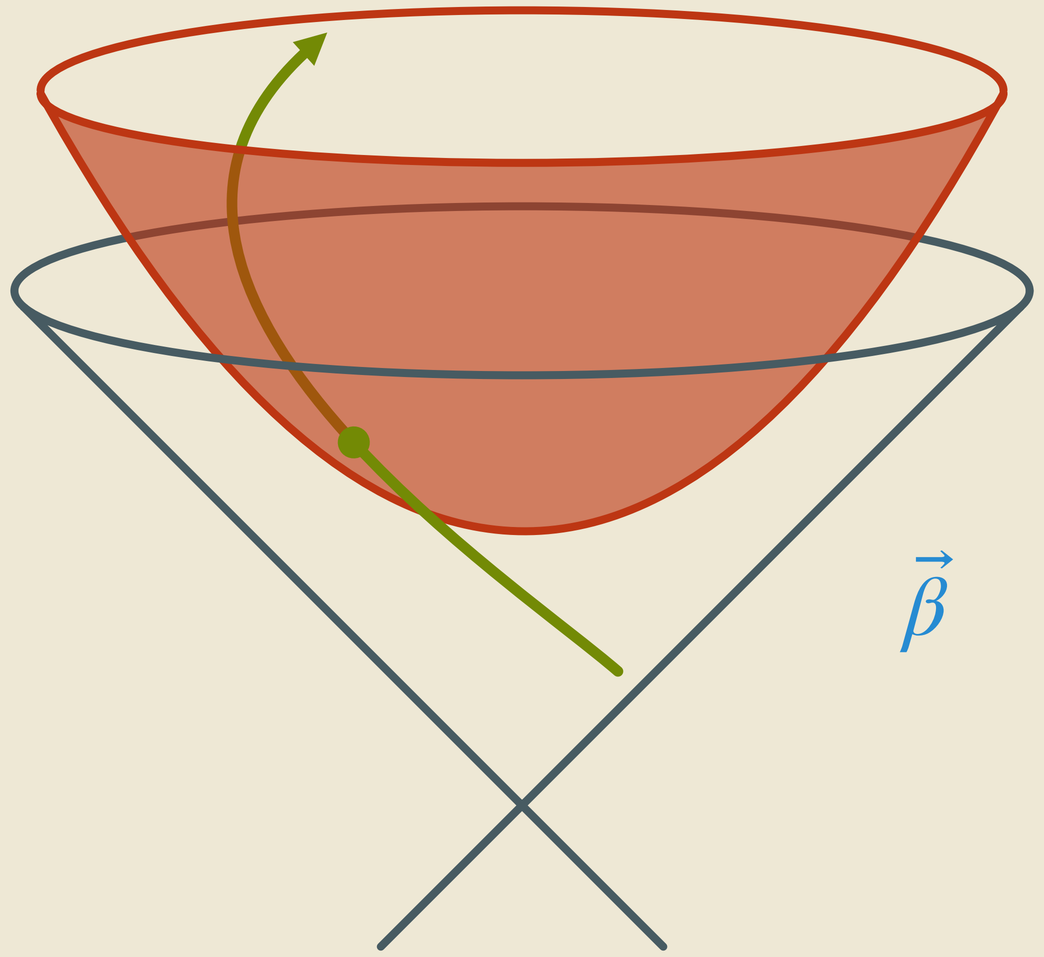
$$ds^2 = -\,e^{-2\alpha}dt^2 + \left(e^{2\beta}\right)_{ij}\mu^i\mu^j$$

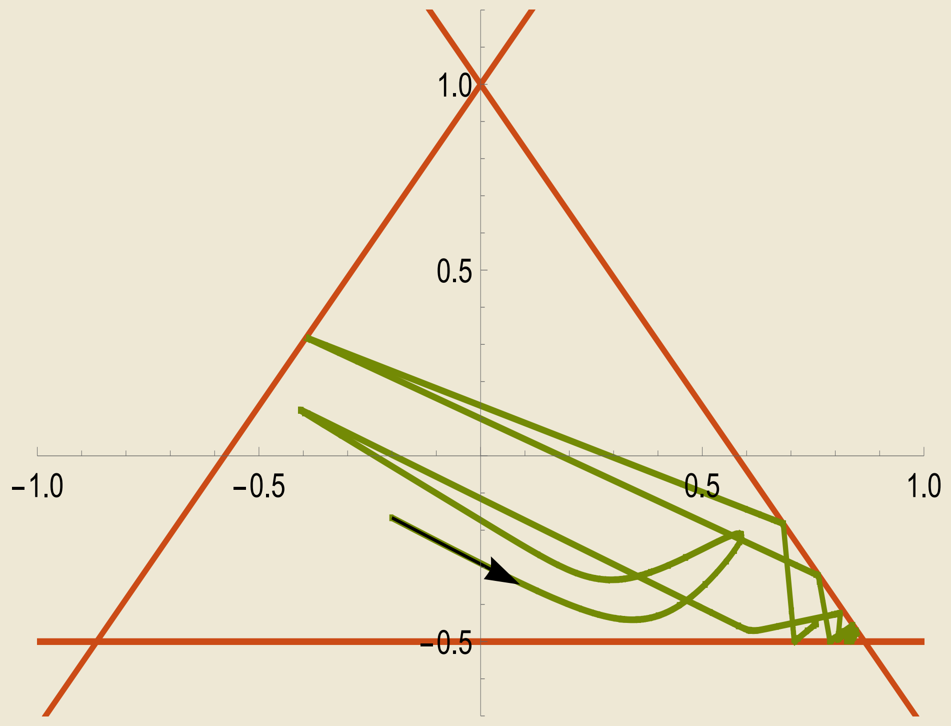
$$R^{(3)} = \frac{1}{2}e^{-2\,\mathrm{tr}\beta}\left(\mathrm{tr}(\beta)^2 - 2\,\mathrm{tr}(\beta^2)\right) \quad \Longrightarrow \quad V(\beta) = e^{4\beta_x} + e^{4\beta_y} + e^{4\beta_z}$$

Potential modifies null geodesics, bounce around in hyperbolic triangle [Misner] [Chitre]

BKL: near spacelike singularities, 4D GR reduces to ultra-local chaotic dynamics of this kind

[Belinskii, Khalatnikov, Lifshitz] [Damour, Henneaux, Nicolai]





Kasner geometries in holography

Planar AdS-RN black hole with charged massive scalar (holographic superconductor)

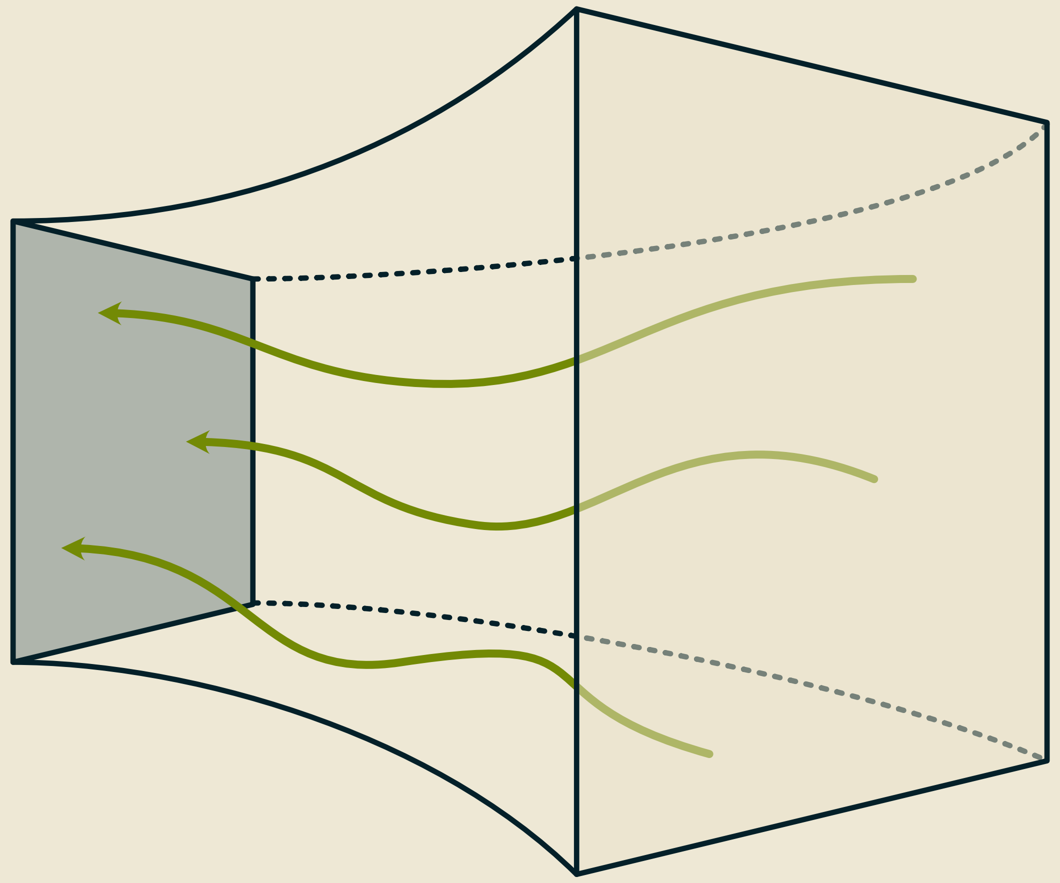
$$S = \frac{1}{2\kappa}\int d^4x\sqrt{-g}\,(R+6) - \frac{1}{2}\int d^4x\sqrt{-g}\left(\frac{1}{2}F^{\mu\nu}F_{\mu\nu} + D^{\mu}\phi\bar{D}_{\nu}\phi + m^2\phi^2\right)$$

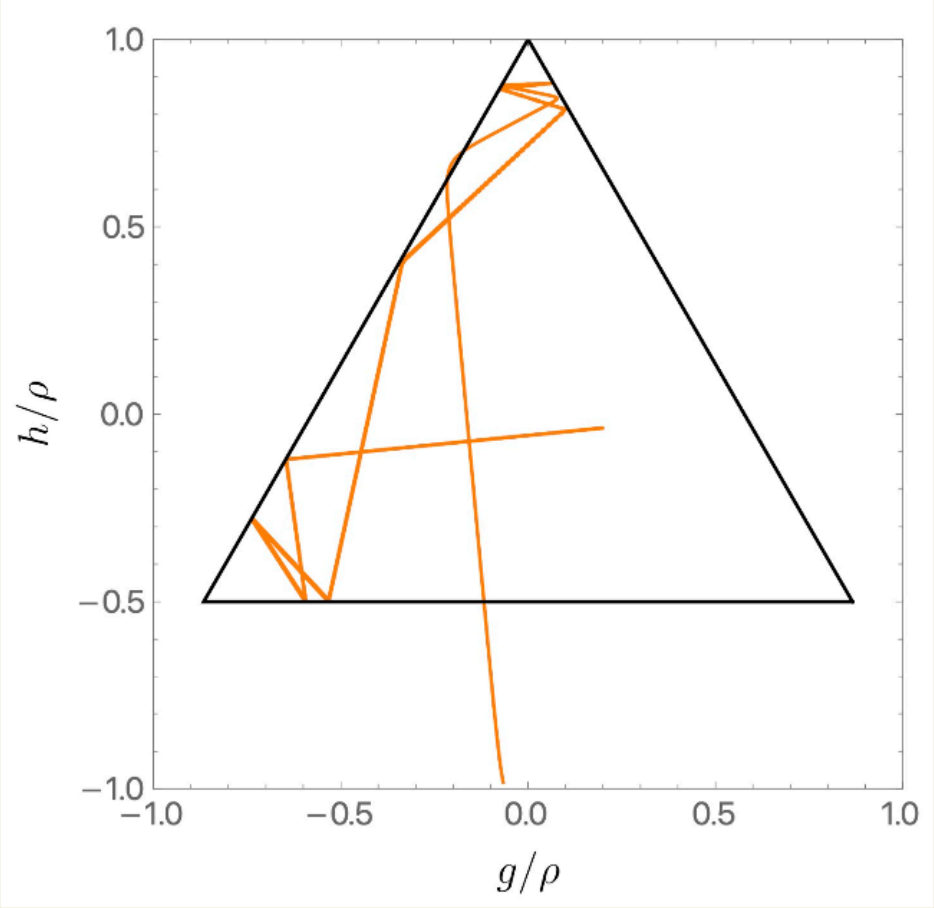
- Nontrivial dynamics behind horizon! [Hartnoll, Horowitz, Kruthoff, Santos]
- Different Kasner epochs, but eventually reaches final state [Henneaux]

Mixmaster-style chaotic behavior obtained from three gauge fields [De Clerck, Hartnoll, Santos]

$$S = \frac{1}{2\kappa}\int d^4x\sqrt{-g}\,(R+6) - \frac{1}{2}\int d^4x\sqrt{-g}\left(\frac{1}{2}F^{\mu\nu}_{(i)}F^{(i)}_{\mu\nu} + \mu^2_{(i)}A^2_{(i)}\right)$$

- Give same hyperbolic triangle dynamics in interior, with AdS asymptotics
- Interpretation in terms of holographic description of black holes?





[De Clerck, Hartnoll, Santos]

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Carroll geometry

From 'relativistic' Lorentz boosts

$$t \to t + \beta x, \qquad x \to x + \beta t$$

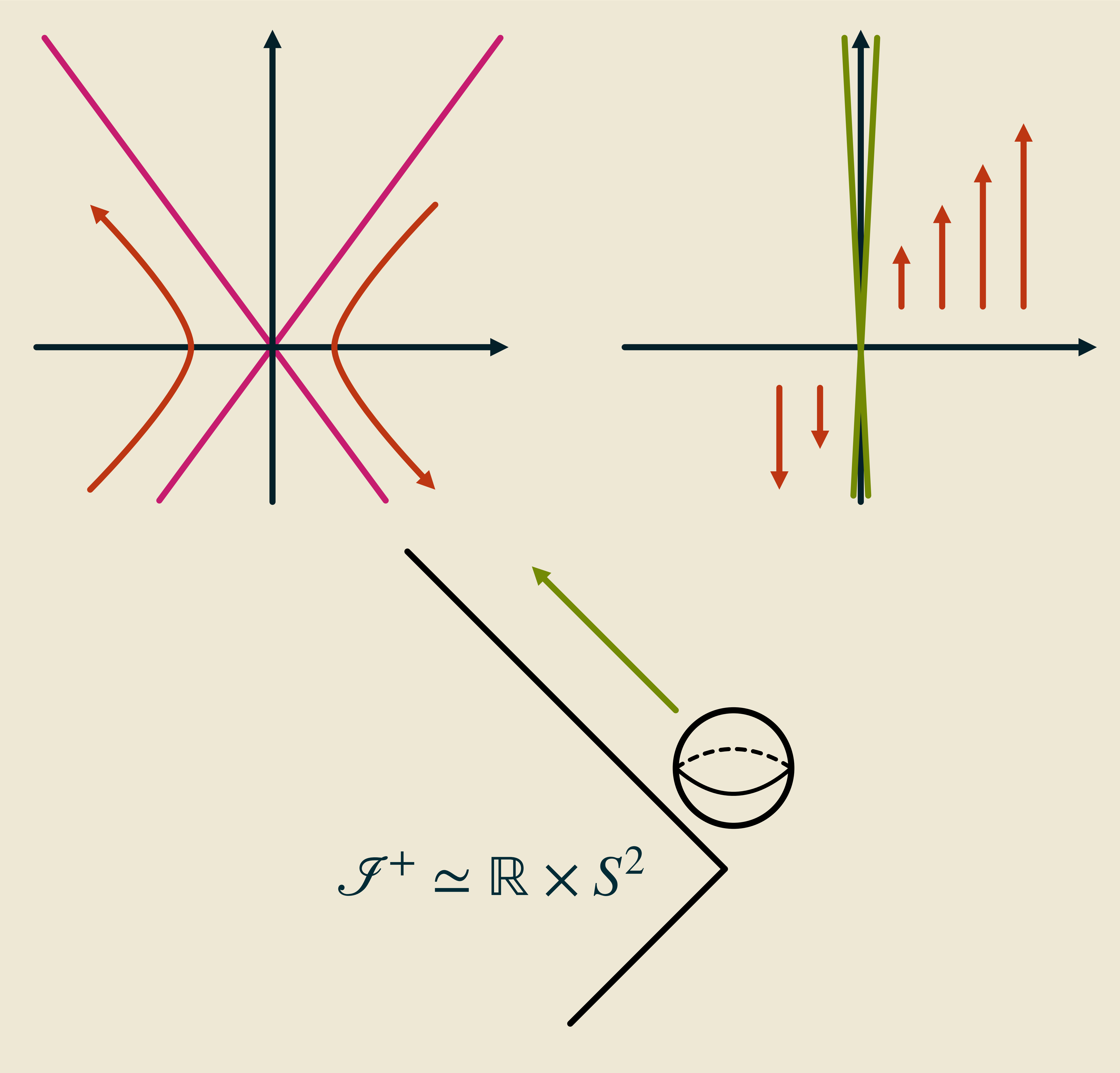
get Carroll boosts in ultra-local $c \to 0$ limit, [Levy-Leblond] [Sen Gupta]

$$t \to t + \lambda x\,, \quad x \to x \quad \text{and} \quad \partial_t \to \partial_t\,, \quad \partial_x \to \partial_x + \lambda \partial_t$$

Less obviously physical than non-relativistic $c \to \infty$ limit, but:

- appears in Lorentzian geometry on null surfaces such as $\mathscr{I}^+$
- BMS asymptotic symmetries are isomorphic to conformal Carroll algebra [see many other talks in this program!]

Here instead use ultra-local limit in bulk to describe BKL dynamics



Newton-Cartan and Carroll geometry

compare: Lorentzian geometry

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -f dt^2 + f^{-1} dr^2 + r^2 d\Omega_2$$

Compatible torsion-free connection $\nabla_\rho g_{\mu\nu} = 0$

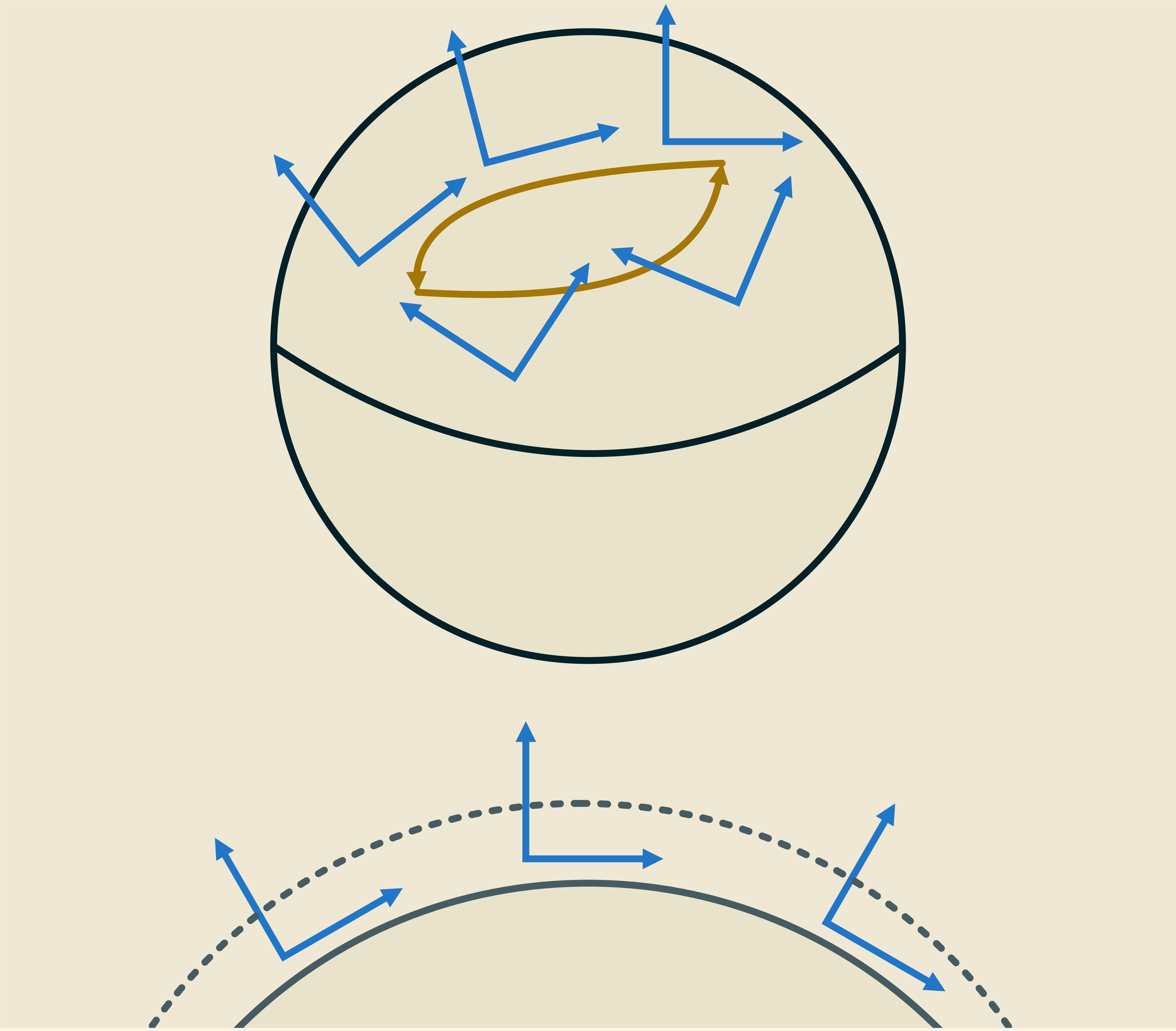
defines curvature $[\nabla_\mu, \nabla_\nu] X^\sigma = -R_{\mu\nu\rho}{}^\sigma X^\rho$

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}_{AB} \begin{pmatrix} \sqrt{f} & 0 \\ 0 & 1/\sqrt{f} \end{pmatrix}_\mu{}^A \begin{pmatrix} \sqrt{f} & 0 \\ 0 & 1/\sqrt{f} \end{pmatrix}_\nu{}^B$$

$$= \eta_{AB}\, e_\mu{}^A e_\nu{}^B$$

metric has local Minkowski structure

Mirror this for local Galilean and local Carroll structures



Newton-Cartan geometry

Galilean boost

$$\begin{pmatrix} t \\ x \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ v & 1 \end{pmatrix} \begin{pmatrix} t' \\ x' \end{pmatrix}$$

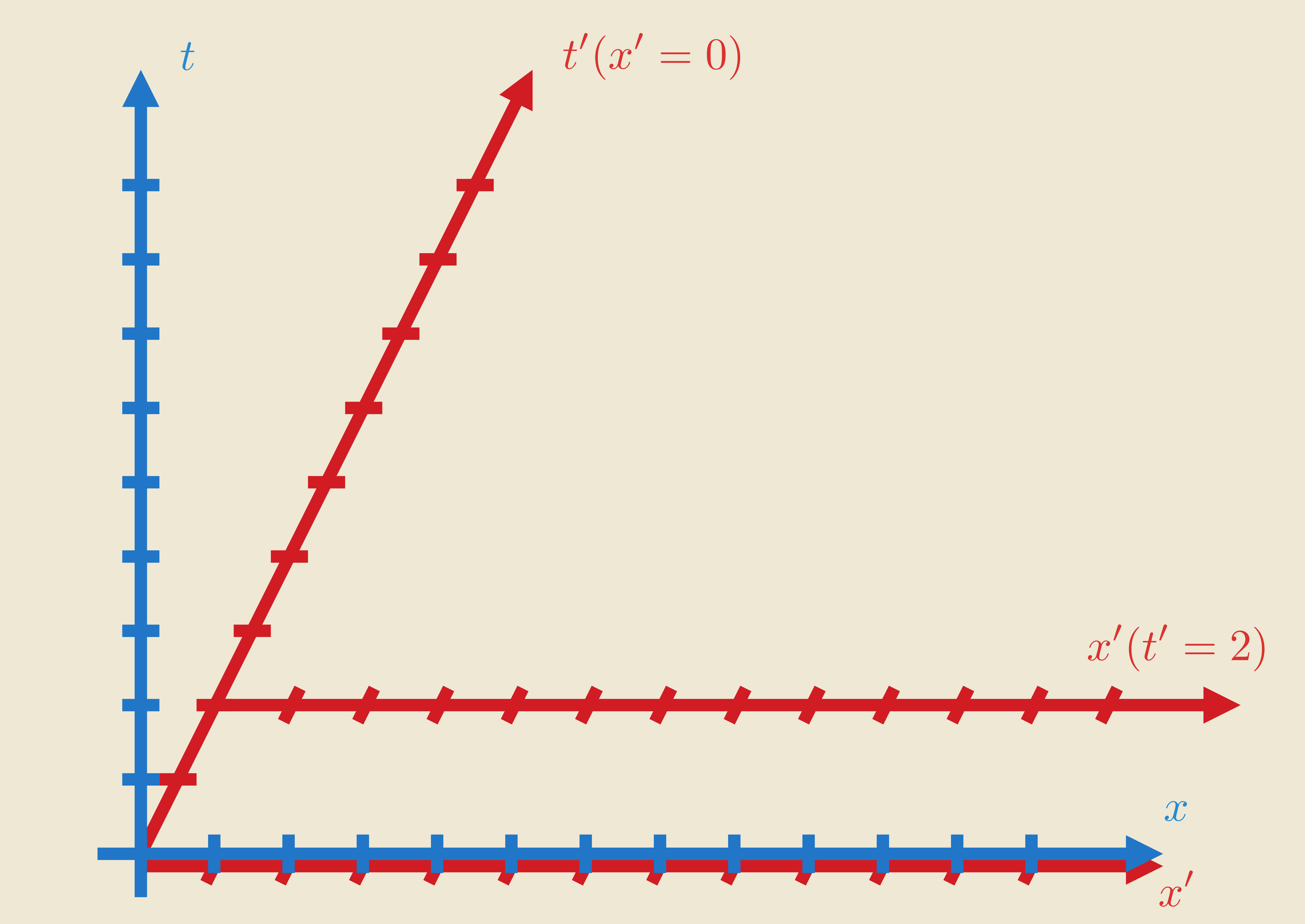
preserves

- time coordinate $\begin{pmatrix} 1 & 0 \end{pmatrix}$
- space direction $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

For curved geometry:

- clock one-form $\tau_\mu dx^\mu \sim \begin{pmatrix} 1 & 0 \end{pmatrix}$
- spatial cometric $h^{\mu\nu}\partial_\mu\partial_\nu \sim$ several $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

known as Newton-Cartan geometry



Newton-Cartan geometry

Newton-Cartan geometry $\tau_\mu(x^\rho)$ and $h^{\mu\nu}(x^\rho)$

Has local Galilean structure!

$$\tau_\mu \sim (1 \quad 0) \quad \text{and} \quad h^{\mu\nu} = \delta^{ab} e^\mu{}_a e^\nu{}_b \sim \text{several} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

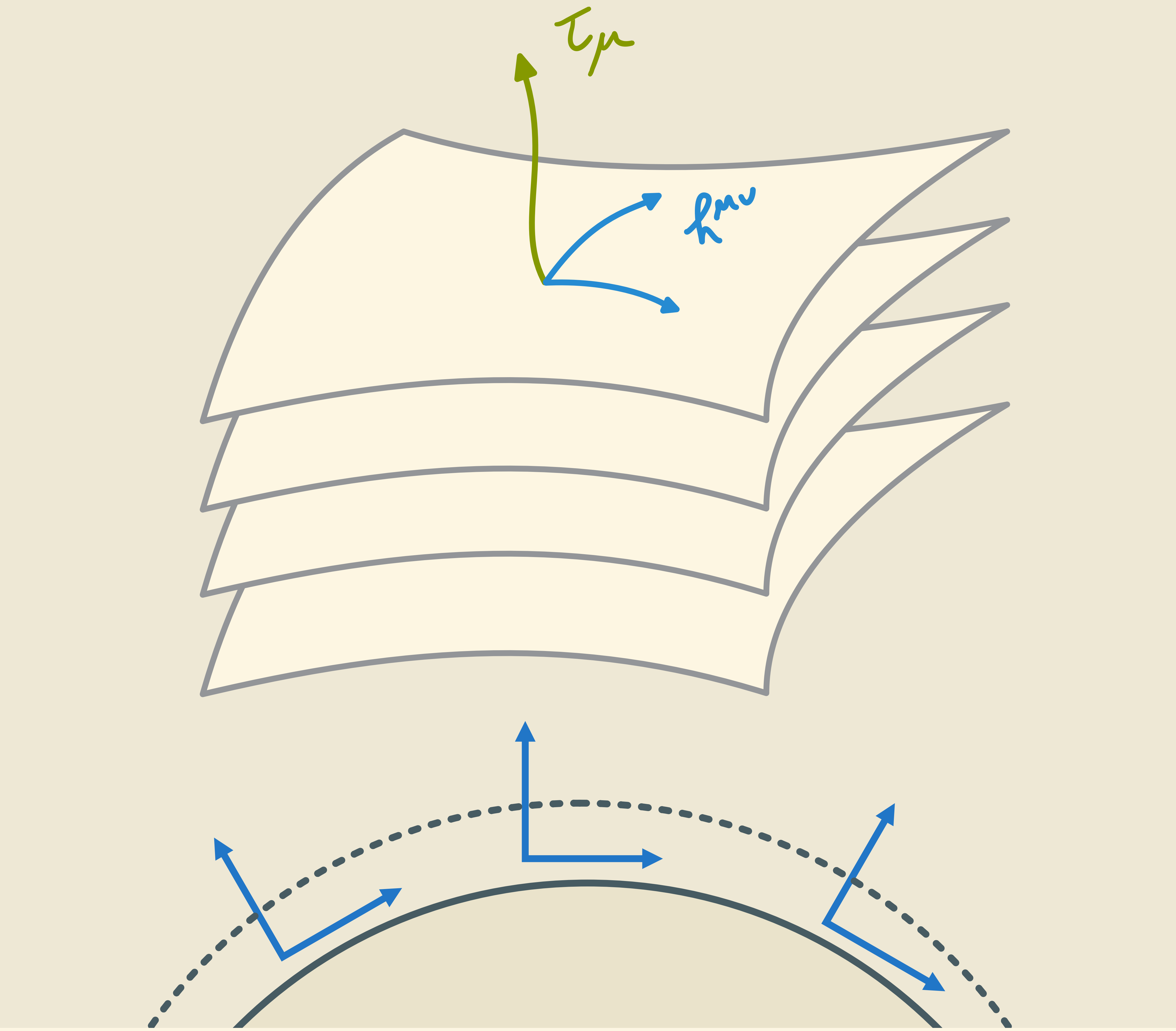
Clock form defines spatial foliation (if $\tau \wedge d\tau = 0$), e.g.

$$\tau_\mu dx^\mu = -\sqrt{1 - \frac{R}{r}} dt\,, \quad h^{\mu\nu}\partial_\mu \partial_\nu = \left(1 - \frac{R}{r}\right) \partial_r^2 + \frac{1}{r^2}\partial_{\Omega_2}$$

Compatible connection $\check{\nabla}_\rho \tau_\mu = 0$ and $\check{\nabla}_\rho h^{\mu\nu} = 0$

curvature $\quad [\check{\nabla}_\mu, \check{\nabla}_\nu]\, X^\sigma = -\check{R}_{\mu\nu\rho}{}^\sigma X^\rho$

torsion $\quad 2\check{\Gamma}^\rho_{[\mu\nu]} = 2\tau^\rho \partial_{[\mu}\tau_{\nu]} \quad$ determined by $d\tau$



Newton-Cartan gravity

Clock one-form $\tau_\mu(x^\rho)$ and spatial cometric $h^{\mu\nu}(x^\rho)$

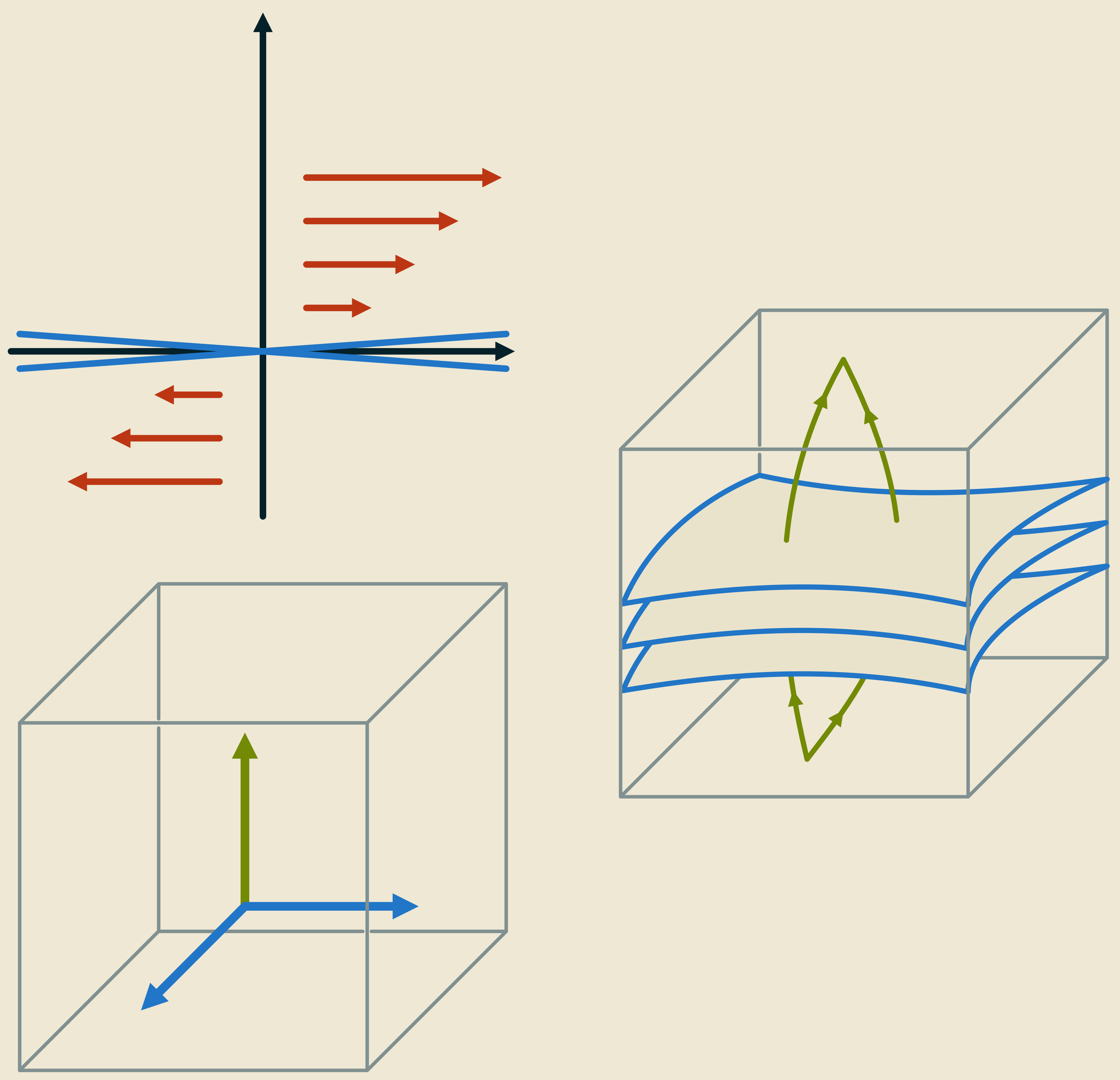
- get *absolute* time if $d\tau = 0$
  - then $\tau = dt$
  - time t is path-independent
- get time dilation if $d\tau \neq 0$!

Dynamical gravity from same ingredients:

- Ricci curvature $\check{R}_{\mu\nu\rho}{}^{\sigma}$
- energy-momentum tensor $\check{T}^{\mu}{}_{\nu}$

Recently: obtain from covariant expansion of GR around $c \to \infty$,
leads to 'type II' Newton-Cartan geometry

[Van den Bleeken] [Hansen, Hartong, Obers] [Bergshoeff, Izquierdo, Ortín, Romano] [Gomis, Kleinschmidt, Palmkvist, Salgado-Rebolledo] [Hartong, Musaeus] …



Carroll geometry

Carroll boost

$$\begin{pmatrix} t \\ x \end{pmatrix} = \begin{pmatrix} 1 & \lambda \\ 0 & 1 \end{pmatrix} \begin{pmatrix} t' \\ x' \end{pmatrix}$$

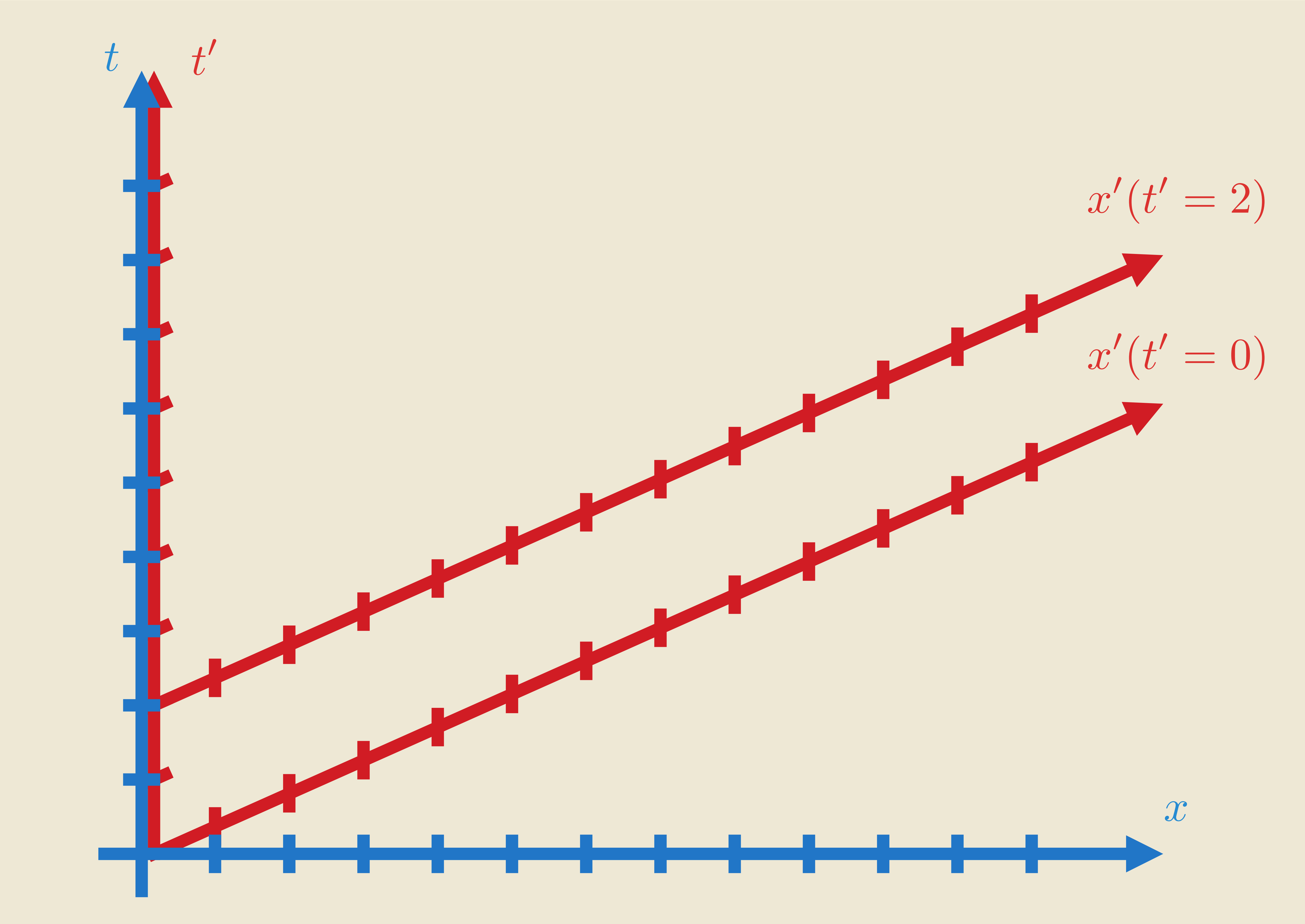
preserves

- time direction $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
- space coordinate $(0 \quad 1)$

For curved geometry:

- time vector field $v^\mu \partial_\mu \sim \begin{pmatrix} 1 \\ 0 \end{pmatrix}$
- spatial metric $h_{\mu\nu} dx^\mu dx^\nu \sim$ twice $(0 \quad 1)$

known as Carroll geometry

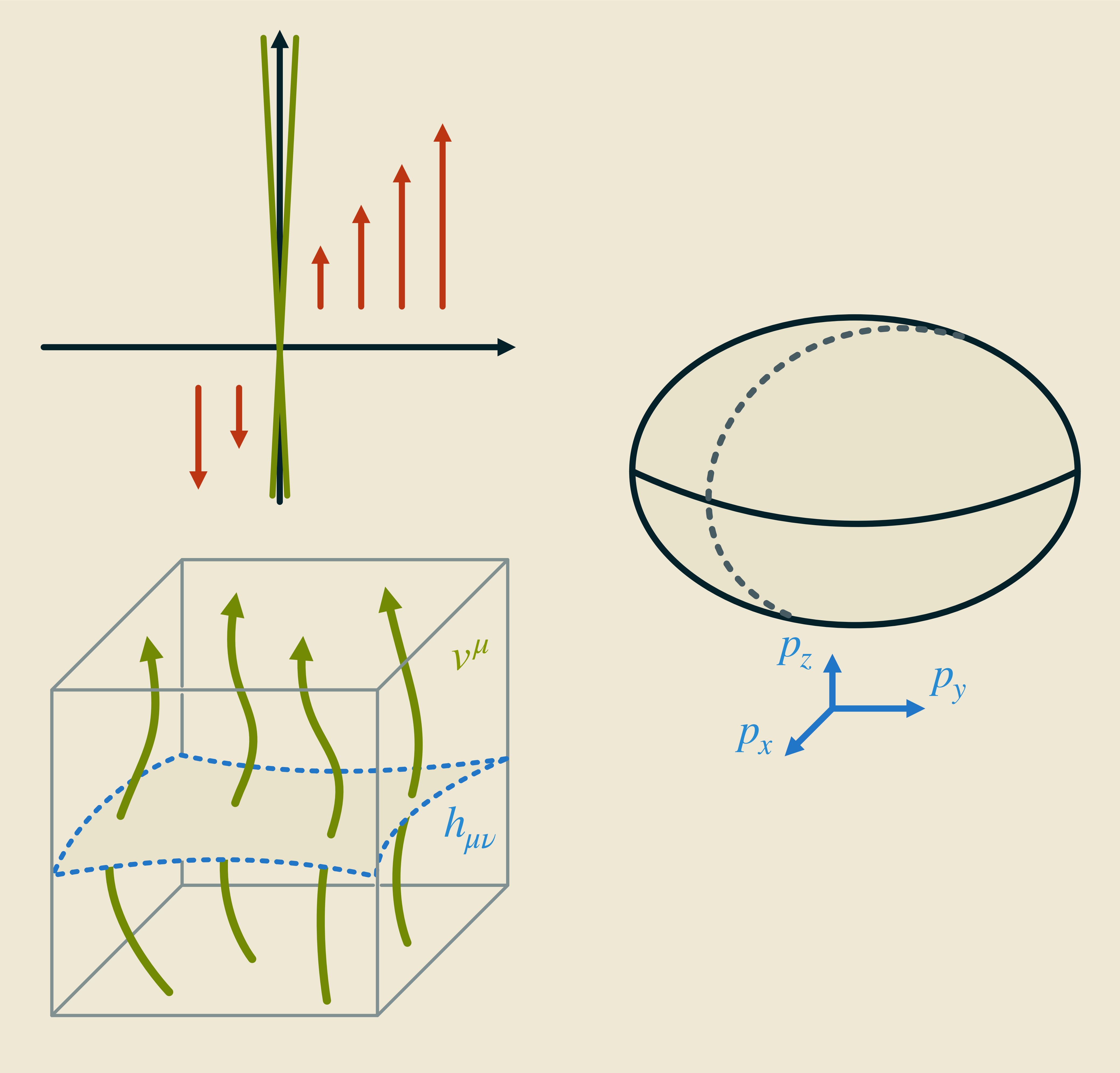


Carroll geometry

Curved Carroll geometry is specified by

time vector field $v^\mu(x^\rho)$ and

spatial 'metric' $h_{\mu\nu}(x^\rho)$



Example: Kasner-type geometry

$$v^\mu \partial_\mu = -\,\partial_t\,, \quad h_{\mu\nu}dx^\mu dx^\nu = t^{2p_x}dx^2 + t^{2p_y}dy^2 + t^{2p_z}dz^2$$

Compatible connection $\tilde\nabla_\rho v^\mu = 0$ and $\tilde\nabla_\rho h_{\mu\nu} = 0$ [other choices too!]

curvature $\quad [\tilde\nabla_\mu, \tilde\nabla_\nu]\, X^\sigma = -\,\tilde R_{\mu\nu\rho}{}^\sigma X^\rho - 2\Gamma^\rho{}_{[\mu\nu]}\nabla_\rho X^\sigma$

torsion $\quad 2\tilde\Gamma^\rho_{[\mu\nu]} = 2h^{\rho\sigma}\tau_{[\mu}K_{\nu]\sigma}$

and extrinsic curvature $K_{\mu\nu} = -\dfrac{1}{2}\mathscr{L}_v h_{\mu\nu}$

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Caroll from Lorentzian

From Lorentzian geometry get Carroll plus corrections by expanding around $c \to 0$

Two-step process [Hansen, Obers, GO, Søgaard]

Rewrite: Choose time vector V^μ and rewrite

$$g_{\mu\nu} = -c^2 T_\mu T_\nu + \Pi_{\mu\nu}\,, \qquad g^{\mu\nu} = -\frac{1}{c^2} V^\mu V^\nu + \Pi^{\mu\nu}$$

exposes overall factors of c^2 in the metric

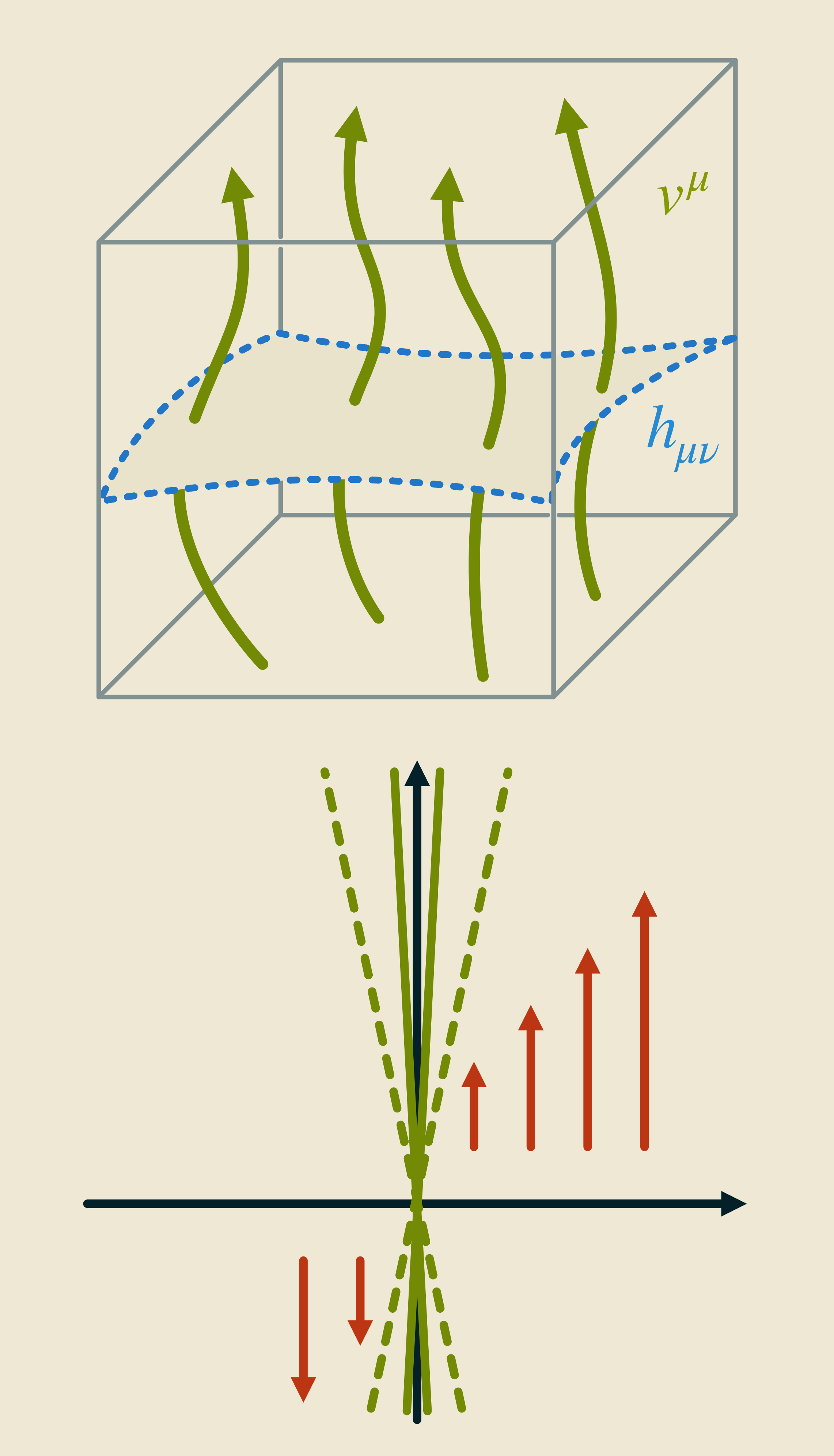
Expand: Carroll geometry at leading order in c^2 expansion

$$V^\mu = v^\mu + c^2 M^\mu + \cdots, \qquad T_\mu = \tau_\mu + \cdots$$

$$\Pi^{\mu\nu} = h^{\mu\nu} + c^2 \Phi^{\mu\nu} + \cdots, \qquad \Pi_{\mu\nu} = h_{\mu\nu} + \cdots$$

local Lorentz boosts → local Carroll boosts + corrections

[Bergshoeff, Izquierdo, Ortín, Romano] [Gomis, Kleinschmidt, Palmkvist, Salgado-Rebolledo]



Caroll from Lorentzian

Carroll connection $\tilde{\Gamma}^{\rho}_{\mu\nu}$ and curvature $\tilde{R}_{\mu\nu\rho}{}^{\sigma}$ from Levi-Civita

Rewrite Levi-Civita with explicit factors of c^2

$$\Gamma^{\rho}_{\mu\nu} = \frac{1}{c^2} S_{(-2)}{}^{\rho}{}_{\mu\nu} + \bar{C}^{\rho}_{\mu\nu} + S_{(0)}{}^{\rho}{}_{\mu\nu} + c^2 S_{(2)}{}^{\rho}{}_{\mu\nu}\,,$$

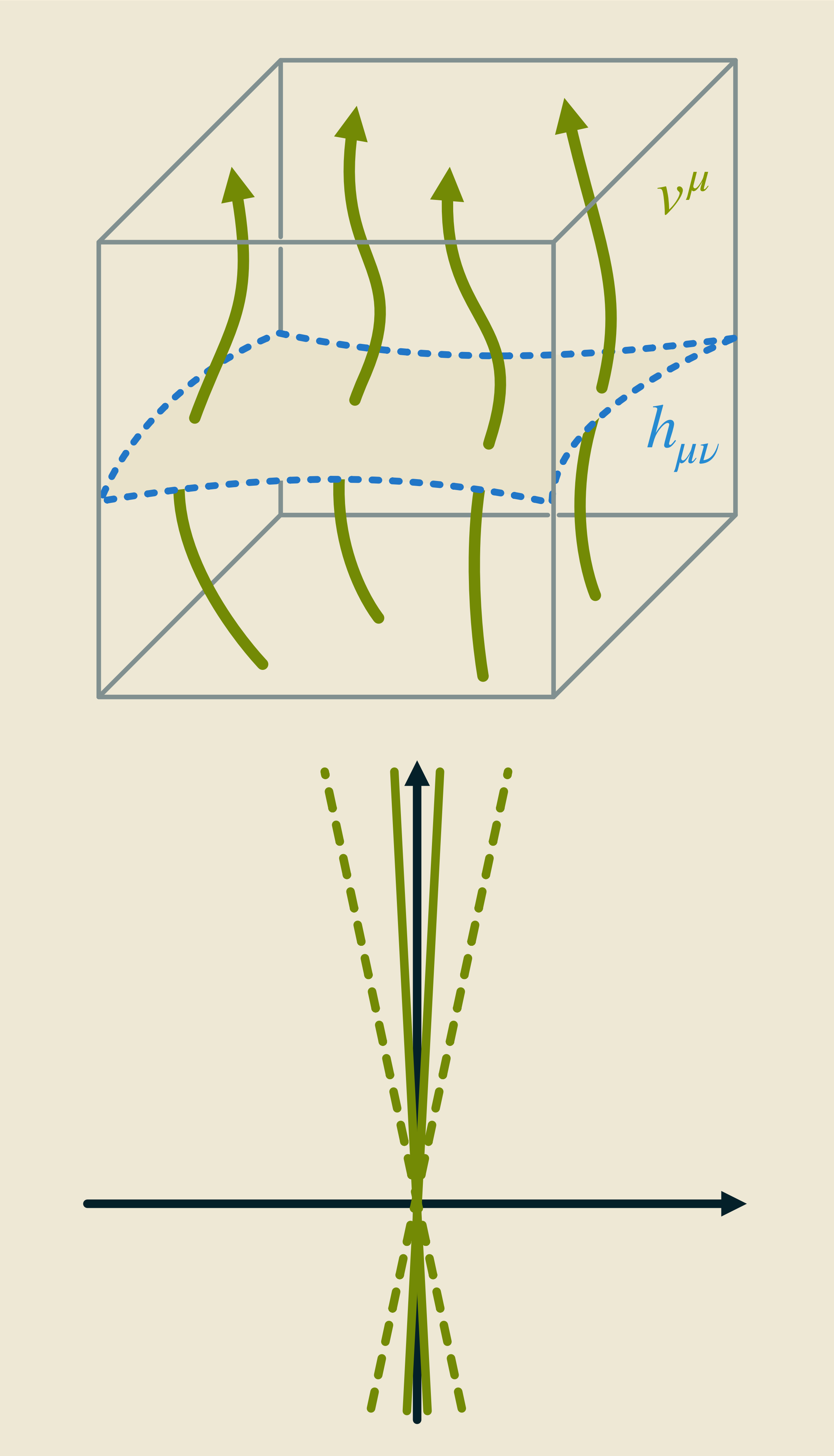
where $S_{(n)}{}^{\rho}{}_{\mu\nu}$ are known tensors. Expand to get $\bar{C}^{\rho}_{\mu\nu} = \check{\Gamma}^{\rho}_{\mu\nu} + \cdots$

Get connection $\tilde{\Gamma}^{\rho}_{\mu\nu}$ so that $\tilde{\nabla}_{\mu} v^{\nu} = 0$ and $\tilde{\nabla}_{\rho} h_{\mu\nu} = 0$ [or any other choice of connection!]

$$\tilde{\Gamma}^{\rho}_{\mu\nu} = -\, v^{\rho}\partial_{(\mu}\tau_{\nu)} - v^{\rho}\tau_{(\mu}\mathscr{L}_{v}\tau_{\nu)} + \frac{1}{2}h^{\rho\sigma}\left(\partial_{\mu}h_{\nu\sigma} + \partial_{\nu}h_{\sigma\mu} - \partial_{\sigma}h_{\mu\nu}\right) - h^{\rho\sigma}\tau_{\nu}K_{\mu\sigma}$$

Non-zero torsion $\tilde{T}^{\rho}{}_{\mu\nu} = 2h^{\rho\sigma}\tau_{[\mu}K_{\nu]\sigma}$

Rewrite $\sqrt{-g} = cE$ where $E = \det(T_{\mu}, \Pi_{\mu\nu})$ and expand $E = e + \cdots$ where $e = \det(\tau_{\mu}, h_{\mu\nu})$



Caroll from Lorentzian

Rewrite the Einstein-Hilbert action, $\mathscr{K}_{\mu\nu} = -\frac{1}{2}\mathscr{L}_V \Pi_{\mu\nu} = K_{\mu\nu} + \cdots$ is extrinsic curvature

$$S = \frac{c^3}{16\pi G}\int_M R\sqrt{-g}\, d^d x$$
$$\approx \frac{c^2}{16\pi G}\int_M \left[\left(\mathscr{K}^{\mu\nu}\mathscr{K}_{\mu\nu} - \mathscr{K}^2\right) + c^2\,\Pi^{\mu\nu}\bar{R}_{\mu\nu} + c^4\,\Pi^{\mu\rho}\Pi^{\nu\sigma}\partial_{[\mu}T_{\nu]}\partial_{[\rho}T_{\sigma]}\right] E\, d^d x$$

From Lorentzian point of view a strange thing to do!

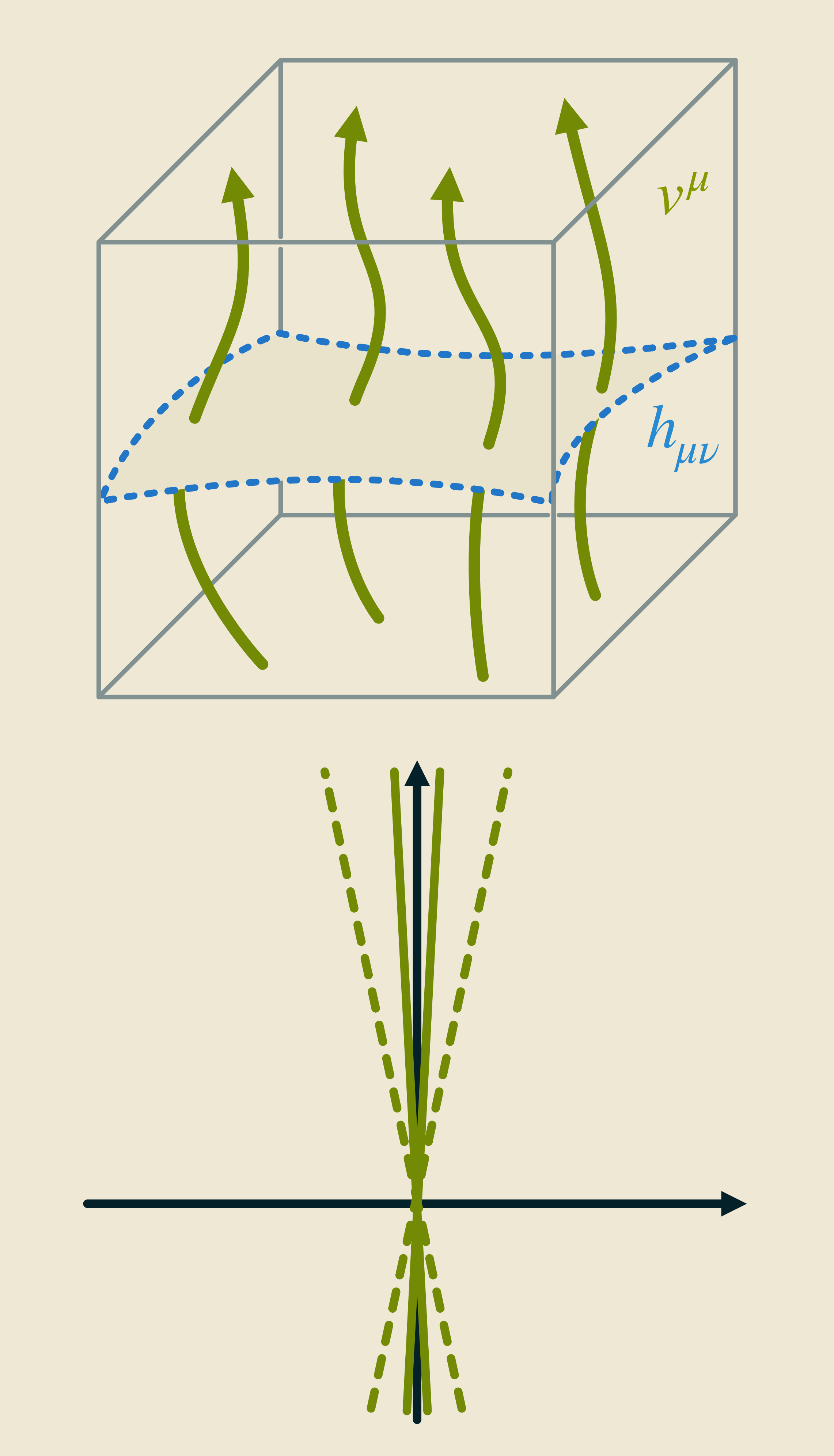
($\bar{C}^{\rho}_{\mu\nu} = \check{\Gamma}^{\rho}_{\mu\nu} + \cdots$ is neither flat nor Lorentz-metric-compatible nor torsion-free)

But enables us to expand the action in c^2, *Carroll geometric expansion!*

$$S = c^2 S_{\mathrm{LO}} + c^4 S_{\mathrm{NLO}} + c^6 S_{\mathrm{NNLO}} + \cdots$$

At leading order get timelike (or electric) Carroll gravity action

$$S_{\mathrm{LO}} = \frac{1}{16\pi G}\int_M \left(K^{\mu\nu}K_{\mu\nu} - K^2\right) e\, d^d x$$



Carroll limit of GR

Expanding GR in $c \to 0$ gives Carroll gravity at LO [Hansen, Obers, GO, Søgaard]

$$S_{\text{EH}} = \frac{c^2}{2\kappa}\int_M \left(K^{\mu\nu}K_{\mu\nu} - K^2\right) e\, d^d x \quad + \cdots$$

similar actions found in [Henneaux] [Hartong] [Henneaux, Salgado-Rebodello]

EOM split into constraint and evolution equations [Hansen, Obers, GO, Søgaard] [Dautcourt]

$$0 = K^{\mu\nu}K_{\mu\nu} - K^2 \qquad\qquad 0 = -R^{(3)} + K^{\mu\nu}K_{\mu\nu} - K^2$$

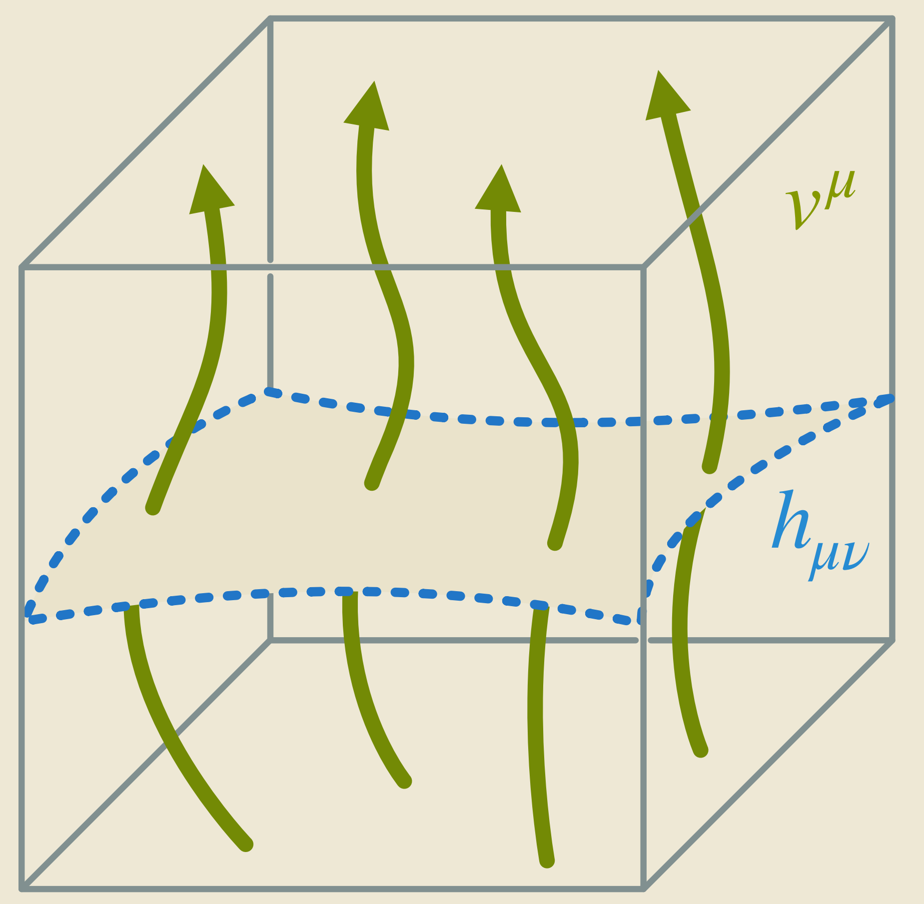
$$0 = h^{\rho\sigma}\tilde{\nabla}_\rho(K_{\sigma\mu} - Kh_{\sigma\mu}) \qquad\qquad 0 = h^{\rho\sigma}\nabla^{(3)}_\rho(K_{\sigma\mu} - Kh_{\sigma\mu})$$

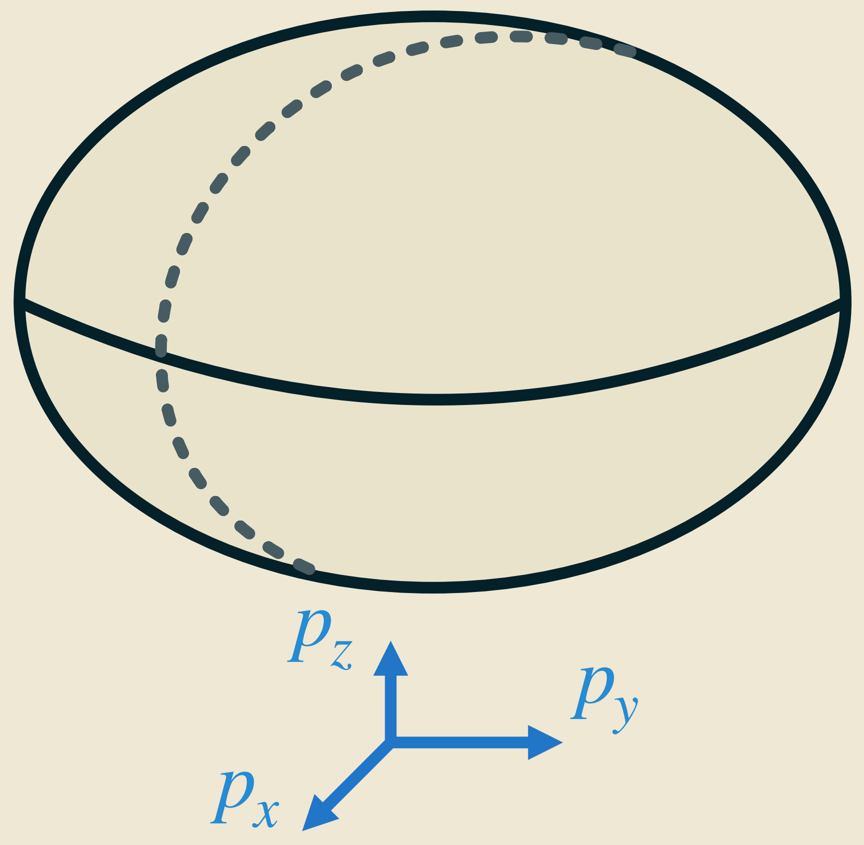
$$\mathscr{L}_v K_{\mu\nu} = -2K_\mu{}^\rho K_{\rho\nu} + KK_{\mu\nu} \qquad\qquad \mathscr{L}_v K_{\mu\nu} = R^{(3)}_{\mu\nu} - 2K_\mu{}^\rho K_{\rho\nu} + KK_{\mu\nu} - \nabla^{(3)}_\mu a_\nu - a_\mu a_\nu$$

Evolution equations are now just *ODEs!*

Solutions include Kasner geometry $\quad v^\mu\partial_\mu = -\partial_t\,,\ \ h_{\mu\nu}dx^\mu dx^\nu = t^{2p_x}dx^2 + t^{2p_y}dy^2 + t^{2p_z}dz^2$

[Henneaux] [Søgaard] [De Boer, Hartong, Obers, Sybesma, Vandoren]





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Kasner solutions in Carroll gravity

Vacuum Carroll LO 'electric' gravity theory has Kasner solutions

$$v^\mu \partial_\mu = -\partial_t\,, \qquad h_{\mu\nu} dx^\mu dx^\nu = t^{2p_x} dx^2 + t^{2p_y} dy^2 + t^{2p_z} dz^2$$

Solves EOM for $S_{\mathrm{LO}} = \dfrac{1}{2\kappa} \displaystyle\int_M \left[K^{\mu\nu} K_{\mu\nu} - K^2 \right] e\, d^d x$

Again can reformulate in terms of β variables

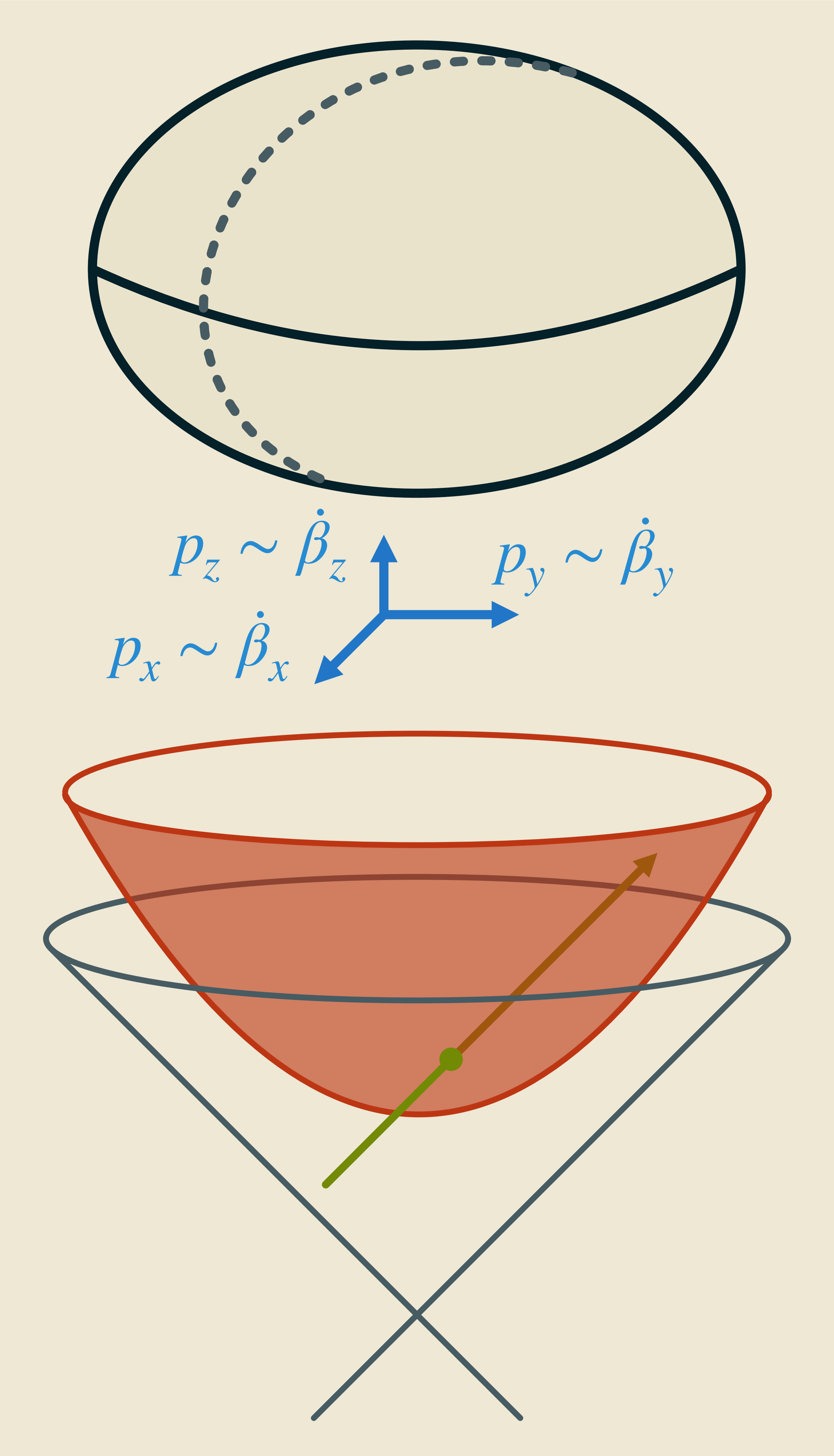
$$v^\mu \partial_\mu = -e^{\alpha(t)} \partial_t\,, \qquad h_{\mu\nu} dx^\mu dx^\nu = \left(e^{2\beta(t)}\right)_{ij} dx^i dx^j$$

Same picture of null geodesics in hyperbolic superspace

$$\dot\beta_i \dot\beta^i = 0\,, \qquad \beta_i(\tau) = \beta_i^{(0)} + v_i \tau$$

How to get richer dynamics? Spatial curvature and/or matter coupling!

Problem: No $R^{(3)}$ in LO theory! No 'gravity walls'!



Mixmaster in LO Carroll gravity

Mimic holographic setup! LO Carroll gravity coupled to electric $U(1)^3$ YM

$$E^{(i)}_\mu = v^\rho F^{(i)}_{\rho\mu}\,, \qquad h^{\mu\rho}h^{\nu\sigma}F^{(i)}_{\rho\sigma} = 0$$

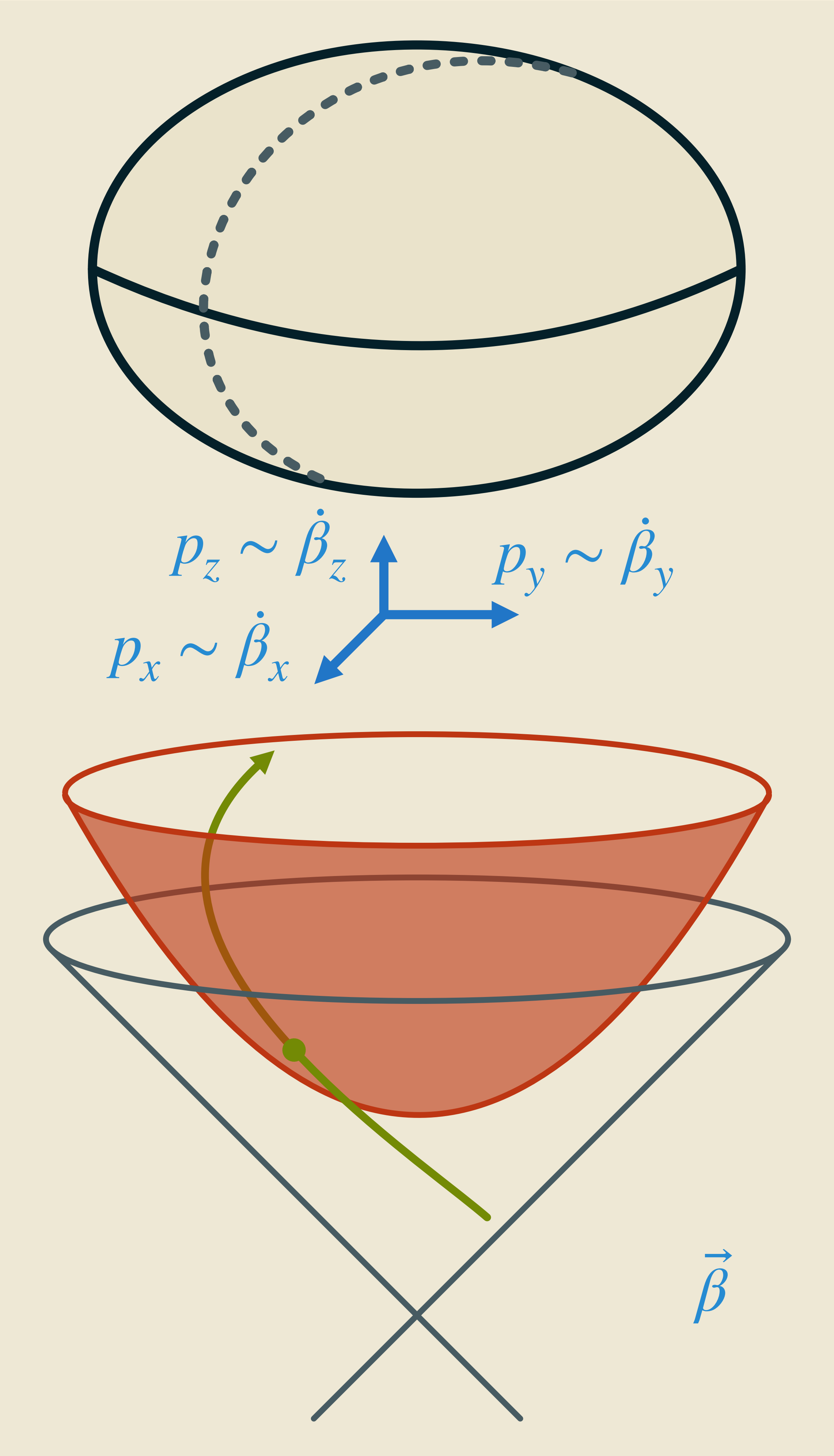
Metric ansatz

$$v^\mu\partial_\mu = -\,e^{\alpha(t)}\partial_t$$
$$h_{\mu\nu}dx^\mu dx^\nu = e^{2\beta_x(t)}\partial_x + e^{2\beta_y(t)}\partial_y + e^{2\beta_z(t)}\partial_z$$

Gauge field ansatz $A^{(i)} = f_i(t)dx^i$ and EOM give

$$E^1 = -\,\phi_x\, e^{-\alpha}e^{\beta_x-\beta_y-\beta_z}\,, \qquad E^2 = -\,\phi_y\, e^{-\alpha}e^{\beta_y-\beta_z-\beta_x}\,, \qquad E^3 = -\,\phi_z\, e^{-\alpha}e^{\beta_z-\beta_x-\beta_y}$$

Will give 'matter walls' for motion of exponents in hyperbolic space



Carroll matter coupling

Adding matter coupling

$$S_G[v,h] + S_M[\phi; v, h]$$

Varying v^μ and $h^{\mu\nu}$ gives metric EOM

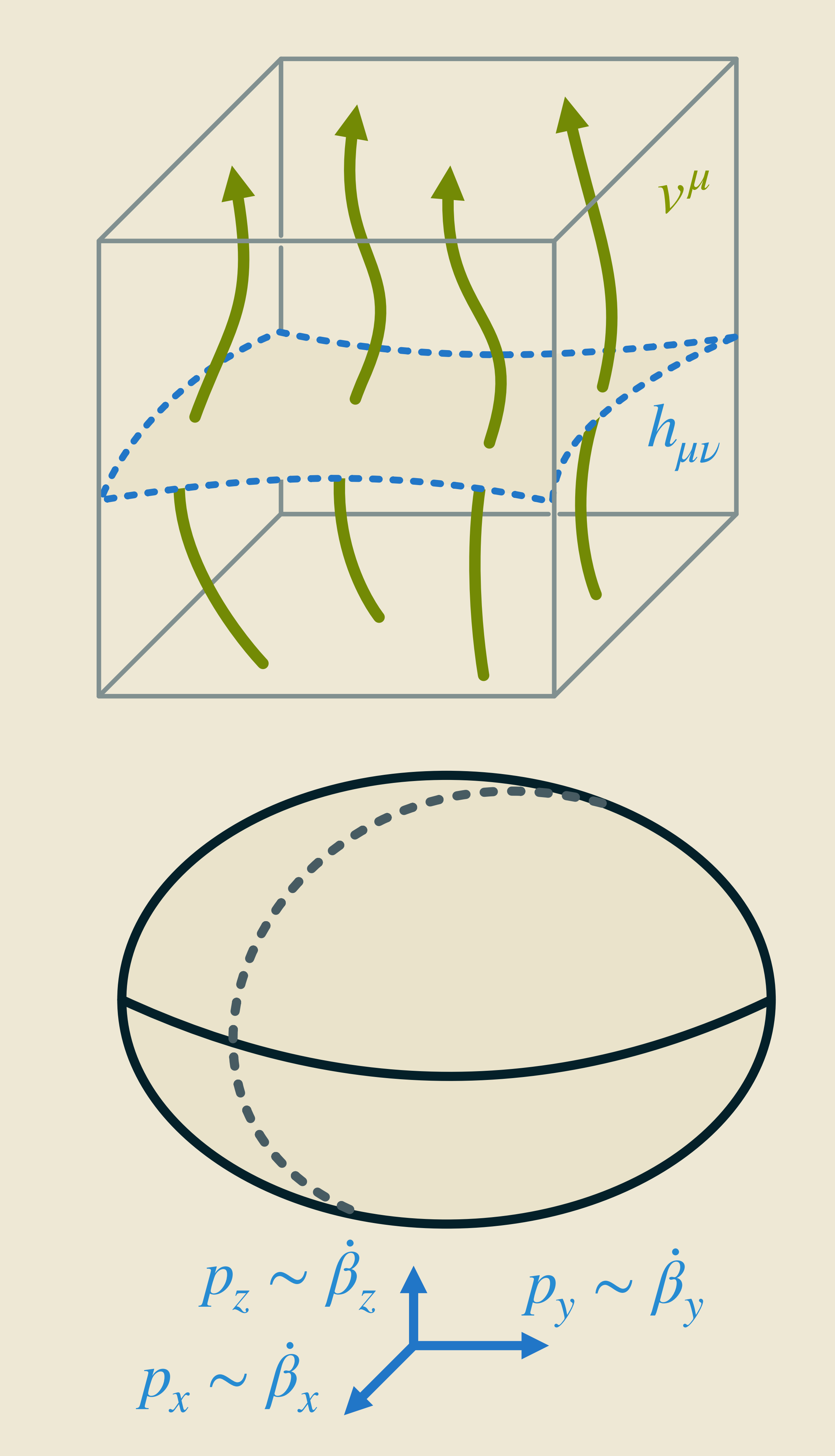
$$0 = \delta S_C + \delta S_M = \frac{1}{2\kappa}\int d^dx\, e\left(2G^v_\mu\,\delta v^\mu + G^h_{\mu\nu}\,\delta h^{\mu\nu}\right) - \int d^dx\, e\left(T^v_\mu\,\delta v^\mu + \frac{1}{2}T^h_{\mu\nu}\,\delta h^{\mu\nu}\right)$$

Resulting LO gravity constraints and evolution equation are

$$\frac{1}{2}\left(K^{\rho\sigma}K_{\rho\sigma} - K^2\right) = \kappa\, v^\mu T^v_\mu$$

$$-h^{\alpha\mu}h^{\rho\nu}\,\tilde{\nabla}_\rho\left(K_{\mu\nu} - Kh_{\mu\nu}\right) = \kappa\, h^{\alpha\mu}T^v_\mu$$

$$\mathscr{L}_v K_{\mu\nu} - KK_{\mu\nu} + 2K_\mu{}^\rho K_{\rho\nu} = -\,\kappa\, h^\alpha_\mu h^\beta_\nu T^h_{\alpha\beta} + \frac{\kappa}{(d-1)}h_{\mu\nu}\left(T^v_\rho v^\rho + T^h_{\rho\sigma}h^{\rho\sigma}\right)$$



Carroll matter coupling

Leading-order Carroll gravity coupled to electric $U(1)^3$ Carroll YM

$$\frac{1}{2\kappa}\int d^d x\, e\left(K^{\mu\nu}K_{\mu\nu} - K^2\right) + \frac{1}{2g}\int d^d x\, e\, h^{\mu\nu}E^{(i)}_\mu E^{(i)}_\nu$$

Leads to sourced constraints and evolution equations

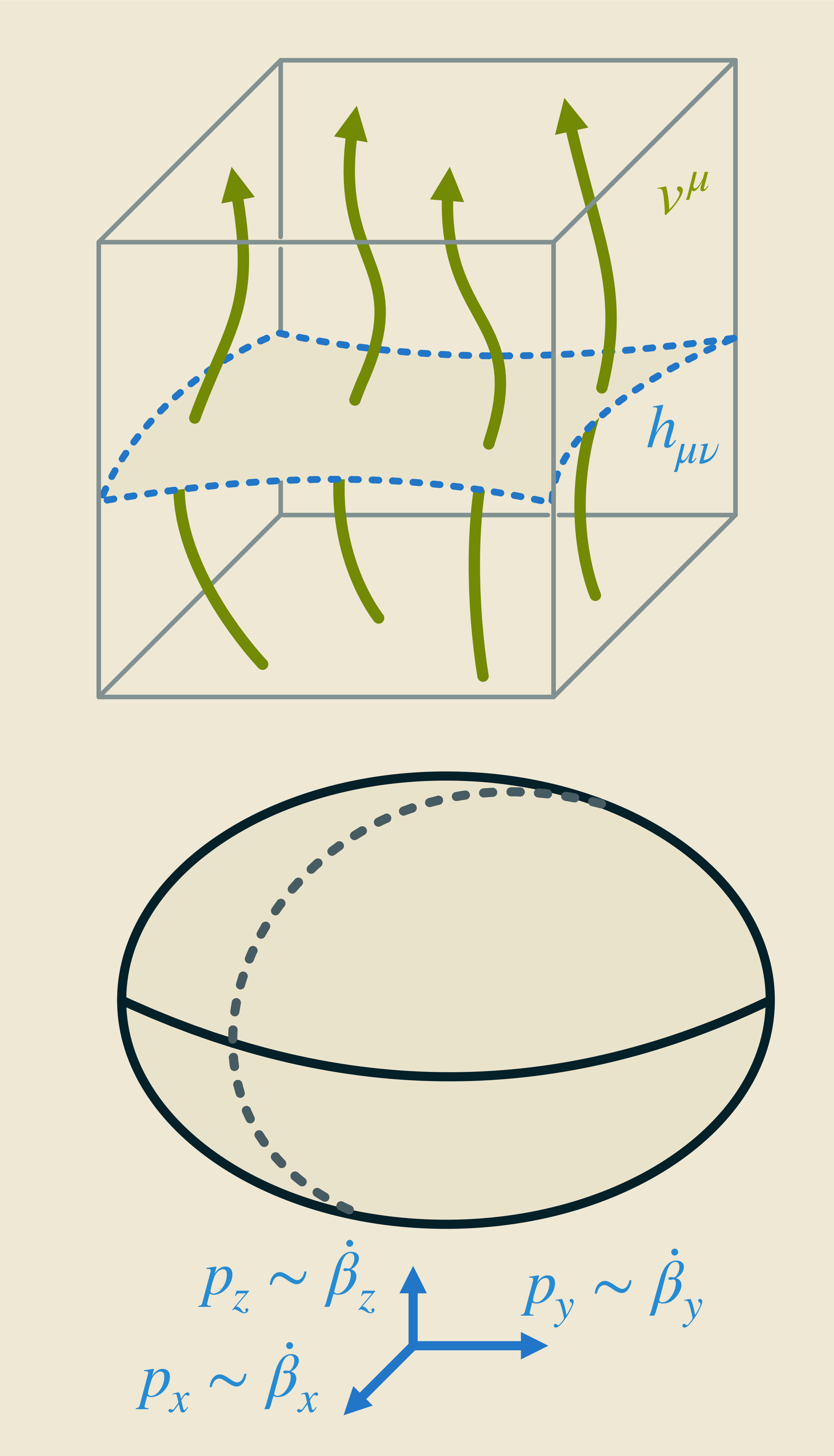
$$\frac{1}{2}\left(K^{\rho\sigma}K_{\rho\sigma} - K^2\right) = -\frac{\kappa}{2g}h^{\mu\nu}E^{(i)}_\mu E^{(i)}_\mu$$

$$-h^{\alpha\mu}h^{\rho\nu}\tilde{\nabla}_\rho\left(K_{\mu\nu} - Kh_{\mu\nu}\right) = 0$$

$$\mathcal{L}_v K_{\mu\nu} - KK_{\mu\nu} + 2K_\mu{}^\rho K_{\rho\nu} = \frac{\kappa}{g}\left(E^{(i)}_\mu E^{(i)}_\nu - \frac{h_{\mu\nu}}{d-1}h^{\rho\sigma}E^{(i)}_\rho E^{(i)}_\sigma\right)$$

together with gauge field EOM

$$\partial_\mu\left(e\, v^{[\mu}h^{\nu]\rho}E^{(i)}_\rho\right) = 0$$



Mixmaster from LO Carroll gravity

Metric ansatz and gauge field

$$v^\mu \partial_\mu = -\, e^{\alpha(t)} \partial_t$$

$$h_{\mu\nu} dx^\mu dx^\nu = e^{2\beta_x(t)} \partial_x + e^{2\beta_y(t)} \partial_y + e^{2\beta_z(t)} \partial_z$$

$$E^1 = -\, \phi_x e^{-\alpha} e^{\beta_x - \beta_y - \beta_z}, \qquad E^2 = -\, \phi_y e^{-\alpha} e^{\beta_y - \beta_z - \beta_x}, \qquad E^3 = -\, \phi_z e^{-\alpha} e^{\beta_z - \beta_x - \beta_y}$$

Sourced constraint equation after shift $\alpha \to \alpha - \mathrm{tr}\beta$

$$\dot\beta^T \begin{pmatrix} 0 & -1 & -1 \\ -1 & 0 & -1 \\ -1 & -1 & 0 \end{pmatrix} \dot\beta = -\left(\dot{\bar\beta}_1\right)^2 + \left(\dot{\bar\beta}_2\right)^2 + \left(\dot{\bar\beta}_3\right)^2 = -\frac{\kappa}{g} V(\beta)$$

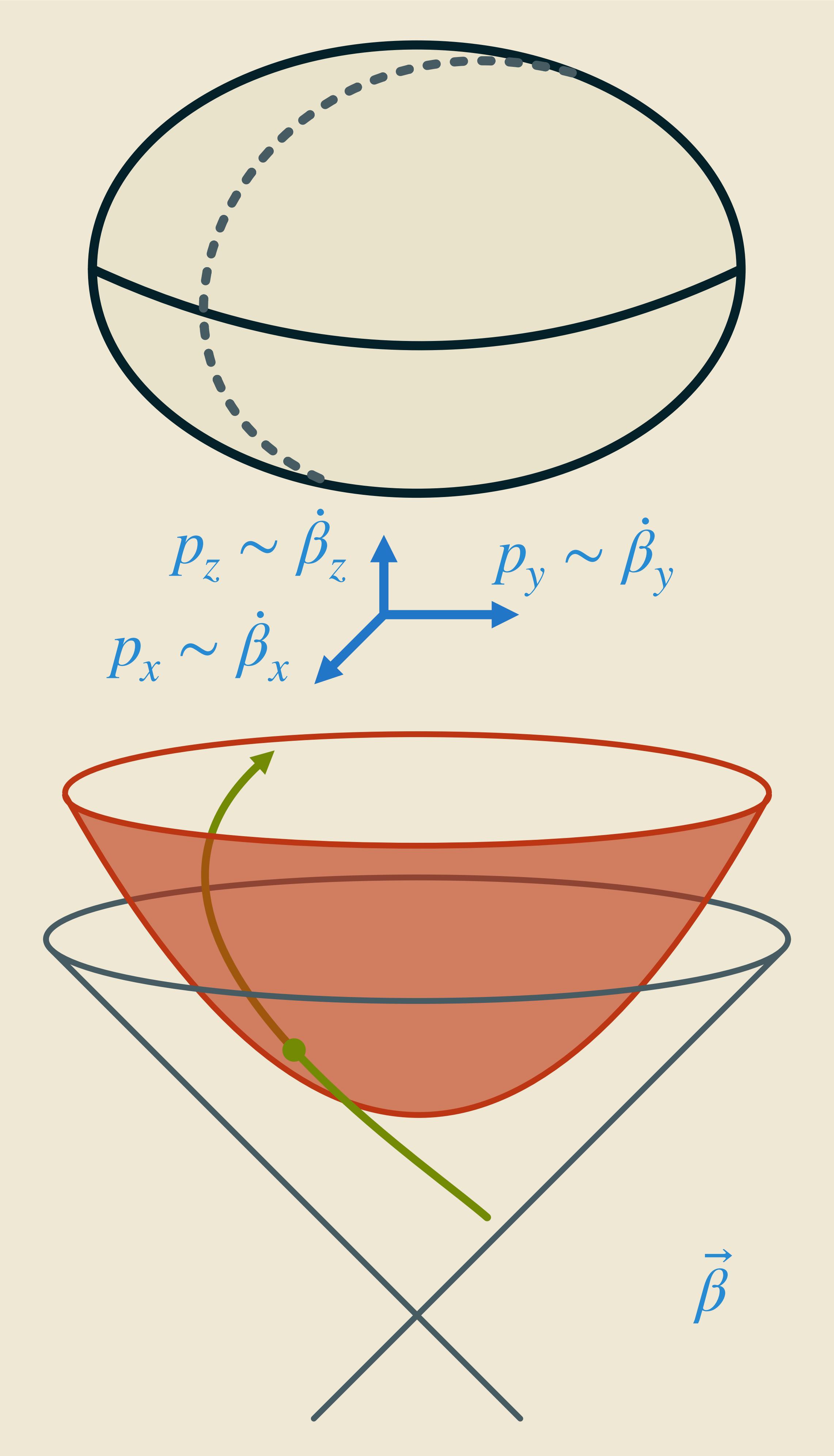
Sourced evolution equation after reparametrization $\tau = \tau(t)$

$$\ddot\beta_i = \frac{\kappa}{2g} \left(1 - \partial_{\beta_i}\right) V(\beta)$$

with potential giving mixmaster triangle motion for exponents in limit

$$V(\beta) = (\phi_x)^2 e^{2\beta_x} + (\phi_y)^2 e^{2\beta_y} + (\phi_z)^2 e^{2\beta_z}$$

$$= (\phi_x)^2 e^{\sqrt{2/3}\left(\bar\beta_0 - \bar\beta_1 - \sqrt{3}\bar\beta_2\right)} + (\phi_y)^2 e^{\sqrt{2/3}\left(\bar\beta_0 - \bar\beta_1 + \sqrt{3}\bar\beta_2\right)} + (\phi_z)^2 e^{\sqrt{2/3}\left(\bar\beta_0 + 2\bar\beta_1\right)}$$



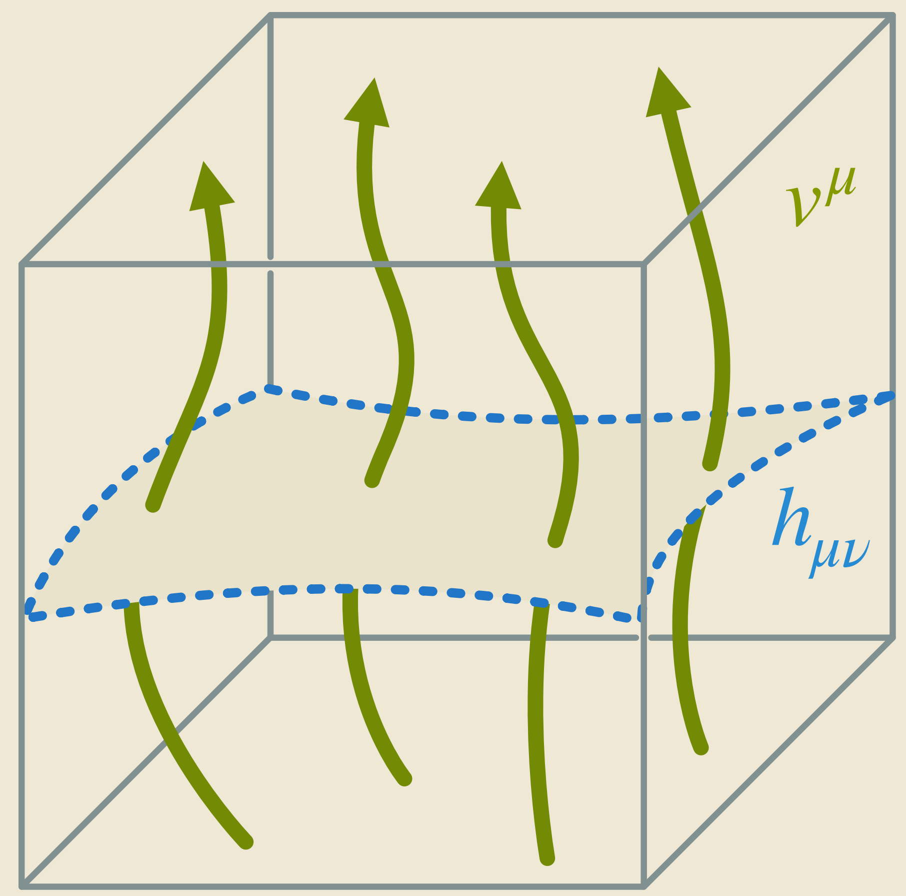
Mixmaster from LO Carroll gravity

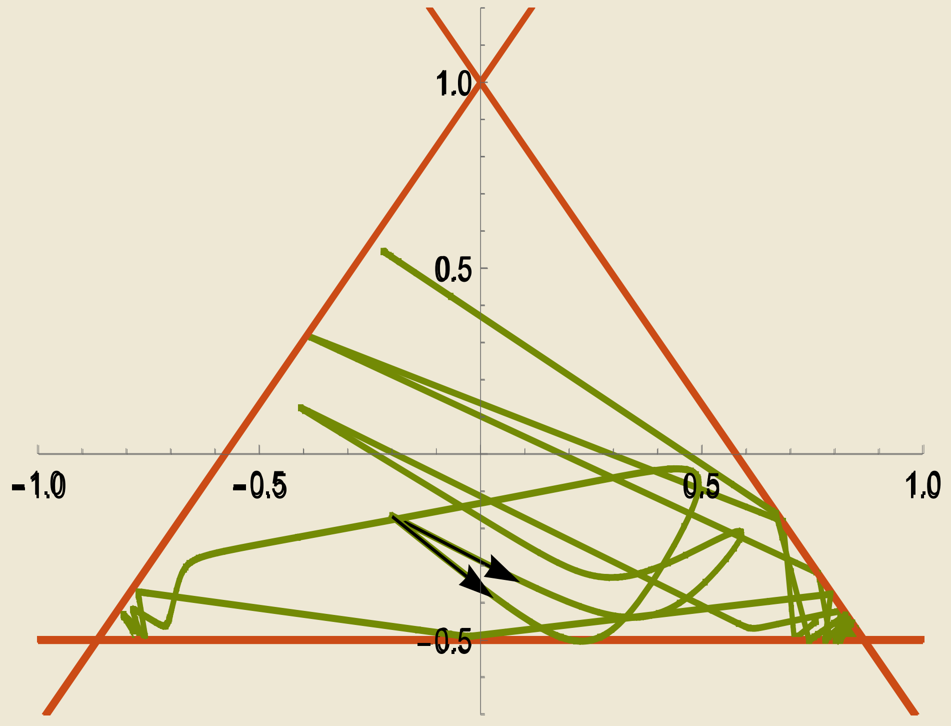
With homogeneous ansatz, EOM of Carroll gravity coupled to $U(1)^3$ gauge fields give

$$\ddot{\beta}_i = \frac{\kappa}{2g}\left(1 - \partial_{\beta_i}\right) V(\beta)$$

where potential gives mixmaster triangle motion for exponents [GO, Pedraza]

$$V(\beta) = (\phi_x)^2\, e^{\sqrt{2/3}\left(\bar{\beta}_0 - \bar{\beta}_1 - \sqrt{3}\bar{\beta}_2\right)} + (\phi_y)^2\, e^{\sqrt{2/3}\left(\bar{\beta}_0 - \bar{\beta}_1 + \sqrt{3}\bar{\beta}_2\right)} + (\phi_z)^2\, e^{\sqrt{2/3}\,(\bar{\beta}_0 + 2\bar{\beta}_1)}$$





[GO, Pedraza]

Reproduce holographic setup of [De Clerck, Hartnoll, Santos '23] using LO Carroll gravity

Generalize it? in LO Carroll gravity evolution equation is always ODE

unlike full GR, still get solvable models even *without spatial homogeneity!*

Explore growth of spatial curvature using tractable models?

Subleading corrections to BKL from Carroll expansion?

Summary and outlook

Close relation between BKL/mixmaster and Carroll expansion of GR

More models easily accessible from Carroll limits/expansion

Off-shell separation of ultralocality limit and strong gravity limit

Work in progress:

- Explicit BKL models with spatial inhomogeneity?
- ‘Standard’ mixmaster from $R^{(3)}$ in NLO Carroll gravity?

Future:

- Subleading corrections to BKL dynamics?
- Tractable (Gowdy) models for ‘spikes’?
- Probes for near-singularity chaos in AdS/CFT?